

Imagine Schools Summer Math Challenge



Grade 6

Dear Imagine Student,

We hope you will enjoy this Math Challenge Packet and work hard to complete all problems on your own or with help from a parent or guardian. All projects in the challenge packet are based on the Common Core State Standards. Therefore, this should be a review for you in some ways, but should stretch you as you apply your understanding of concepts you learned throughout this past year. We suggest doing one project each day. Once you have finished with the project you select for the day, try to find a way to discuss it with a friend, parent, or relative. Think about how the skills and concepts in the problem you completed are connected to other things in your home, environment, or daily routine. Find ways to apply your new understanding to real world situations.

Math is all about problem solving. One of the best ways to learn math is to try out problems in which you have to devise your own strategy to find the solution. There is usually more than one way to solve math problems. While working on the problems in this packet, you may discover shortcuts and use your own process or set of rules to calculate or determine the appropriate solution. Make sure to keep notes, include your work so you can justify your solutions. In other words, be sure you can answer the question, "How do you know?" Explaining how you arrived at an answer immediately tells others what you're learning along the way.

Sincerely,
Imagine Schools National Academic & Character Team

Project #1

Domain: Ratios and Proportional Relationships

Standards:

6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

6.RP.1.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Directions:

1. Solve the following problem.
2. Show all steps you take to find your solution.

The ratio of the number of boys to the number of girls at school is 4:5.

1. What fraction of the students are boys?
2. If there are 120 boys, how many students are there altogether?

Project # 2

Domain: Ratios and Proportional Relationships

Standards:

6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

6.RP.3a. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Directions:

1. Find five examples of ratios in the real world. Write them down and describe the situation in which they are found. *Remember, ratios are comparisons of two quantities which can be written in the following ways:

1) a to b

2) $\frac{a}{b}$

3) a : b

Example: At the grocery store, Amanda noticed that there were three times as many carts as there were baskets for shoppers to use to carry their food. The ratio of carts to baskets (c : b) is 3 to 1.

2. Create a problem using ratios for your parents/guardians or friends to solve. Write both your problem and solution below.

Project # 3

Domain: The Number System

Standard:

6.NS.4.: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.

Directions:

1. Read the problem below.
*Use the hints to help if needed to solve the riddle.

I'm thinking of two numbers. Their greatest common factor is 6. Their least common multiple is 36. One of the numbers is 12. What is the other number?

Hints:

- 1) What are the factors of 12?
- 2) Can the other number be 3? Why or why not? The number cannot be 3 because the greatest common factor of both of the numbers is 6. (3 cannot have a factor).
- 3) Can the other number be 6? Why or why not?

Show your work!

2. Create two riddles like this one to challenge your friends or parents/guardians. One of the two should include a negative integer or fraction. Create 2-3 hints for each to help in case your problem-solvers need some assistance.

Project # 4

Domain: The Number System

Standards:

6.NS.1.: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

6.NS.3.: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Directions:

1. Complete the problem below.

Chef Julius Grayson had an empanada recipe that called for $\frac{3}{4}$ lb onions and $1\frac{1}{2}$ lbs of pork. He was preparing the recipe for a special event and needed to quadruple it to make enough for all of his guests. How many pounds of onions and pounds of pork would he need for the recipe?

Show all of your work.

2. Create a problem about the estimated cost of ingredients for the large event recipe if onions cost \$2.99/lb and pork costs \$5.49/lb. Include both an estimated solution and a true solution to check to see that your estimation is reasonable.



Project #5

Domain: The Number System

Standards:

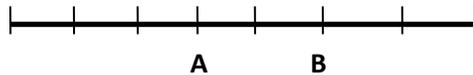
6.NS.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

6.NS.6.c.: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Directions:

1. Read the problem below.

A number line from 0 to 1 is divided into seven equal segments. What fraction names point A? What fraction names point B?



2. Draw your own number line on graph paper to show twelve equal segments from 0 to 1. Label intervals on the number line to show the fractional units that represent each section.

3. Create a second number line and plot the following points on it:

- | | |
|------------|-------------------------|
| a. -2 | d. the opposite of zero |
| b. 3 | e. 2.5 |
| c. $-(-4)$ | f. $-1/2$ |

Project #6

Domain: The Number System

Standard:

6.NS.5. Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation.



Directions: Follow the directions to complete the exercise below.

1. Use an integer to represent 25 feet below sea level
2. Use an integer to represent 25 feet above sea level.
3. What would 0 (zero) represent in the scenario above?
4. Draw a map or sketch to illustrate the above scenario and label each the designated regions.

Project #7

Domain: The Number System

Standards:

6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.

6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.



Directions:

1. Read the following problem and answer the questions.
2. Draw a table or pictures to help you solve. *Show all of your work in the space below or on a separate sheet of paper.

A runner ran 20 miles in 150 minutes. If she runs at that speed,

1. How long would it take her to run 6 miles?
2. How far could she run in 15 minutes?
3. How fast is she running in miles per hour?
4. What is her pace in minutes per mile?

Project #8

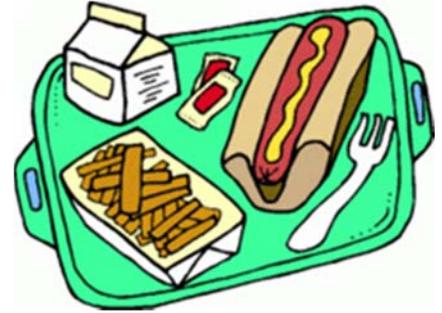
Domain: The Number System

Standard:

6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

1. List all the factors of 48.
2. List all the factors of 64.
3. What are the common factors of 48 and 64?
4. What is the greatest common factor of 48 and 64?
5. Use the greatest common factor and the distributive property to find the sum of 36 and 8.
6. Solve:

The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how many days will both schools serve pizza again?



Project #9

Domain: The Number System

Standard:

6. NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Directions:

Solve the following problem:

The florist can order roses in bunches of one dozen and lilies in bunches of 8. Last month she ordered the same number of roses as lilies. If she ordered no more than 100 roses, how many bunches of each could she have ordered?

What is the smallest number of bunches of each that she could have ordered?

Use the space to create a table or draw a diagram to solve the problem.
Be careful to explain your reasoning.



Project #10

Domain: Expressions & Equations

Standard:

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.

Directions:

Solve the following. Show all of your work.

1. What is the value of:
 - a. 0.2^3
 - b. $5 + 2^4 \times 6$
 - c. $7^2 - 24 \div 3 + 26$

2. What is the area of a square with a side length of $5x$?

3. $6^x = 1,296$

4. Write an algebraic expression for each of the following:
 - 7 less than 3 times a number

 - 3 times the sum of a number and 5

 - 7 less than the product of 2 and a number

 - Twice the difference between a number and 5

 - The quotient of the sum of x plus 4 and 2

Project #11

Domain: Expressions & Equations

Standard:

6.EE.2.c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Directions:

1. Evaluate the following expression when $x = 4$ and $y = 2$

$$\frac{x^2 + y^3}{3}$$

2. Solve the following:

It costs \$100 to rent the skating rink plus \$5 per person. Write an expression to find the cost for any number (n) of people.

Then use your formula to find the cost for 25 people to skate.

3. Create an equation of your own for a parent or friend to solve. Write your solution and show all of your work below.

Project # 12

Domain: Expressions & Equations

Standard:

6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*

Directions:

Solve the following. Show all of your work.

- Given that the width of the flower beds is 4.5 units and the length can be represented by $x + 2$, write an equation to express the area of the flowers below.



- Write out an explanation for each part of the problem below:
 - Use your understanding of multiplication to interpret $3(2 + x)$.
 - Use a model to represent x , and make an array to show the meaning of $3(2 + x)$.
 - Draw an array to represent the equation: $3(2+x)$
- Prove that $y + y + y = 3y$

Project # 13

Domain: Expressions & Equations

Standard:

6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, ending on the purpose at hand, y number in a specified set.



Directions:

1. Complete both problems below.

- A. Billy was offered a job at the nearby golf course. The owner offered him \$500.00 per seven day week or \$50 the first day and agreed to double it for each following day. How could Billy make the most amount of money?

Which deal should he accept and why?

- B. Sally is having a birthday party with 10 people. When everyone gets there she asks everyone to introduce themselves and shake everyone's hand. How many handshakes will there be? How do you know? Show your work below.

2. Create three real-world mathematical problems involving variables to represent unknown numbers. *Be sure to create an answer key with explanations of how to solve each of your problems.

Project #14

Domain: Expressions & Equations

Standard:

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

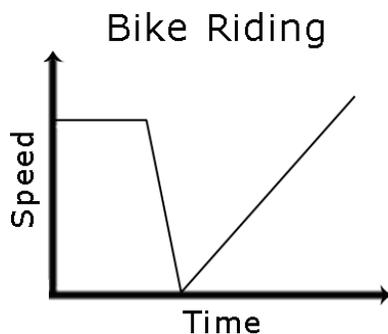
Directions:

1. What is the relationship between the two variables (shown in the table below)?
Write an expression that illustrates the relationship.

| | | | | |
|-----|-----|---|-----|----|
| x | 1 | 2 | 3 | 4 |
| y | 2.5 | 5 | 7.5 | 10 |

2. Solve the problem below.

Describe the change in speed over time shown by the graph.



3. Make up two new problems of your own to show your understanding of how to analyze change.

Project #15

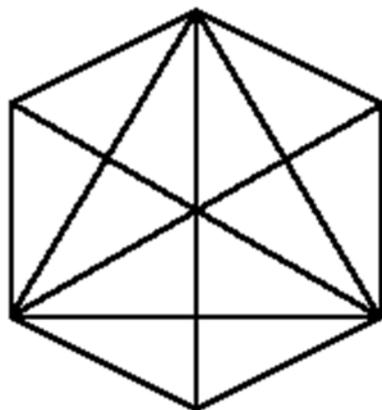
Domain: Geometry

Standard:

6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Directions:

1. Study the figure below.
2. Answer the questions and complete the activities under the figure.



Questions:

- How many triangles are in this figure?
 - How many parallelograms are in this figure?
3. Break apart the geometric figure into its base components. In other words, on a separate piece of paper, draw every possible triangle and every possible parallelogram you identified and counted to answer questions 1 & 2.
 4. Then color the shapes and put them back together to form a variety of new shapes.

Project # 16

Domain: Geometry

Standard:

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Directions:

Part I

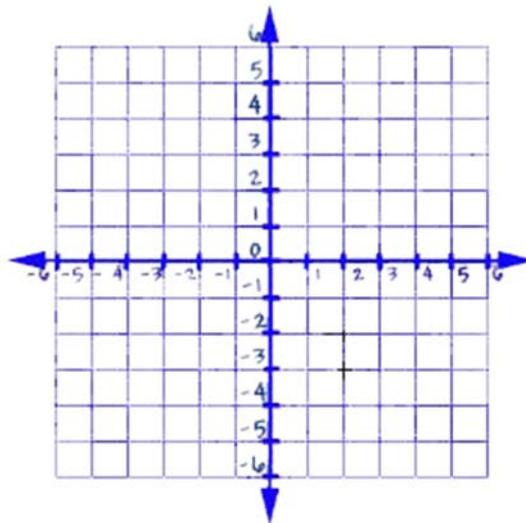
1. Use the following coordinates to draw polygons on the coordinate plane below.

- A. (2, -3)
- B. (-2, -1)
- C. (-3, 4)
- D. (1, 3)

Name the figure: _____

- A. (-5, -3)
- B. (-4, -6)
- C. (4,-4)

Name the figure: _____



Part II

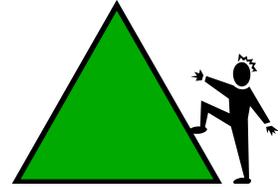
2. On graph paper, draw your own coordinate plane. Label the X and Y axes.
3. Choose a room in your house and study the arrangement of the furniture.
4. Measure the dimensions of at least four pieces of furniture in the room you chose (i.e., your bed, desk, and chest of drawers if you chose your bedroom).
5. Create a scale, then graph the pieces of furniture on your graph paper coordinate plane.
6. Write directions using your coordinate plane and furniture model to give to a parent to see if they can complete a transformation of the furniture according to the directions and scale model you created.

Project # 17

Domain: Geometry

Standard:

6. G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.



Directions:

Solve each problem below using your knowledge of how to find the area of rectangles and triangles.

1. Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.
2. Find the area of the trapezoid shown below using the formulas for rectangles and triangles.
3. A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Project #18

Strand: Statistics and Probability

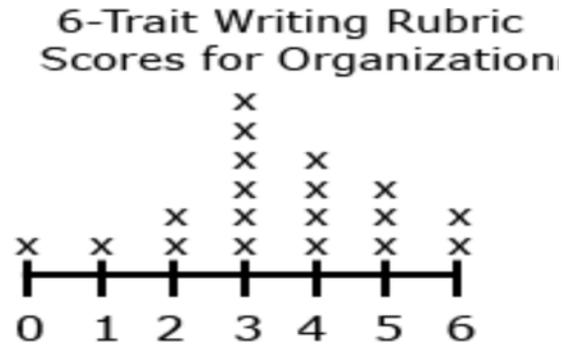
Standard:

6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Directions:

Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

- How many students are represented in the data set?
- What are the mean and median of the data set?
- What do these values mean? How do they compare?
- What is the range of the data? What does this value mean?



Project #19

Domain: Statistics & Probability

Standard:

6.SP.5. Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Directions:

Solve the following. Show all of your work and explain how you arrived at your solutions. Use charts, graphs or diagrams to help you.

Problem 1:

Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below:

Project 1: 18

Project 2: 15

Project 3: 16

Project 4: ??

What does she need to make on Project 4 so that the average for the four projects is 17?

Explain your reasoning.

Problem 2:

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data. Use this information to solve the problem below.

The following data set represents the size of 9 families:

3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Extensions

The following projects are based on standards you will learn in seventh grade. They should be challenging for you at this point. Try them out and save your work along the way.

Project #20

Domain: The Number System

Directions:

1. Solve the problem below.
2. Create three-dimensional models by making real pizza pies and take pictures of your creations or create models with representative objects.



Pizza Party:

How many different types of pizza can you make with the following toppings: pepperoni, tomatoes, bacon, onions and green peppers? Show your answer.

The chef at Pinelli's Pizza just told you that they can create pizzas with two different kinds of pizza on the same pie. For example, you could have a half pepperoni and half bacon pizza. How does this affect the number of types of pizzas you created? Will the number of options double? Why or why not? Show all of your work below.

Hint: To add to the fun of this problem, create real or pretend pizzas and use pennies, nickels, dimes, erasers, M&Ms, Reese's Pieces, gumdrops, or other objects to create your models and figure out your solution.

Project #21

Domain: Expressions and Equations

Directions:

1. Read the problem and study the data on the chart below.
2. Convert the fuel amounts to gallons, then solve.



Derek started his car (automatic shift) drove 9 km and spent 3 minutes waiting at traffic lights. About how much gasoline did Derek's car use?

CHART GIVEN:

AUTOMOBILE FUEL USAGE

Starting .015 Liters

Idling for 1 minute .047 Liters

Moving

Manual Shift 30km 3.0 Liters

Automatic Shift 27km 3.0 Liters

Converting Liters to Gallons

1 US Gallon = 3.7854118 Liters

Project #22

Domain: Various

Directions:

1. Develop a survey like the example below. Include at least 3 groups for the survey (e.g. 3 rounds).
2. Once you have conducted 3 rounds of the survey, evaluate the actual results of your survey in comparison to your prediction following the 2nd round of the survey.



Example:

Present the following question to 10 people:

Which of the following 3 is your favorite sport: Soccer, Basketball, and Baseball/Softball.

Then, use your collected results and percentages to predict what the number of responses will be in your 3rd group of 10.

Which of the following is your favorite sport: Soccer, Basketball, or Baseball (Softball, for a girl)?

1) Record your results from the 1st and 2nd rounds of survey in the table below:

| Soccer | Basketball | Baseball/Softball |
|---------------|-------------------|--------------------------|
| 6/10 = 60% | 2/10 = 20% | 2/10 = 20% |

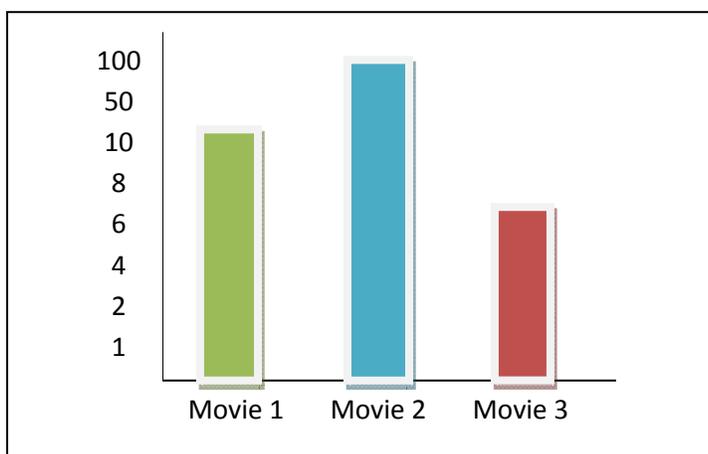
Project #23

Domain: Various

Directions:

1. Create a list of popular movies from the past five years. *You may need to use online resources to find this information.
2. Survey 25 to 30 people and ask each person to pick his or her favorite movie from a list of top 10 movies you created from the past five years.
3. Record the results on a frequency table.
4. Draw a bar graph of the data that could be misleading.

Example:



5. Draw a second bar graph that depicts the information clearly.
6. Next, find the ticket sales (in millions) for each of the movies on your list. *You will need to use online resources to find this information.
7. Find the mean, median, range and mode of the ticket sales. You may use a calculator to complete these computations.
8. Show all your work and final results.

Project #24

Domain: Various

Directions:

1. Solve the following problems below.

1. Being a problem solver is something like being a detective! A detective has to solve crimes by guessing what happened and checking the guess to see if it fits the situation. For some problems, your best strategy may be to make a guess and then check to see if your answer fits the problem. If not, decide if your guess was too high or too low and then make a second "guesstimate." A good detective keeps records (usually some kind of chart) to help see any patterns and to narrow down the possibilities. You should do this too. The results of incorrect guesses can give you valuable clues to the correct solution. Guess and then check the solution to this problem:

Use exactly 50 coins to make one dollar. You must have at least one penny, one nickel, one dime, and one quarter.

2. Team managers predict a crowd of about 2500 for Friday's football game. About how many packages of cups should the concession stand manager plan to order, if the cups come five dozen to a package? Explain how you estimated the number of packages.
3. If a square pyramid is placed on top of a cube, how many faces, vertices, and edges will the new geometric solid have? (Assume the square bottom of the pyramid is the same size as a face on the cube.) Illustrate. Create a model of the 3D figure.
4. Two squares are beside each other. One square has 6 times the length of the other square, how many times greater is the area of the larger square? How do you know? Draw both squares using a ruler and prove your results.
5. Pet Parade:
Mr. James has 14 total pets made up of cats, dogs and guinea pigs. What are all the combinations he could have?