

Imagine Schools' Summer Math Challenge



Grade Seven

Dear Imagine Student,

We hope you will enjoy this Math Challenge Packet and work hard to complete all problems on your own or with help from a parent or guardian. All projects in the challenge packet are based on the Common Core State Standards. Therefore, this should be a review for you in some ways, but should stretch you as you apply your understanding of concepts you learned throughout this past year. We suggest doing one project each day. Once you have finished with the project you select for the day, try to find a way to discuss it with a friend, parent, or relative. Think about how the skills and concepts in the problem you completed are connected to other things in your home, environment, or daily routine. Find ways to apply your new understanding to real world situations.

Math is all about problem solving. One of the best ways to learn math is to try out problems in which you have to devise your own strategy to find the solution. There is usually more than one way to solve math problems. While working on the problems in this packet, you may discover shortcuts and use your own process or set of rules to calculate or determine the appropriate solution. Make sure to keep notes, include your work so you can justify your solutions. In other words, be sure you can answer the question, "How do you know?" Explaining how you arrived at an answer immediately tells others what you're learning along the way.

Sincerely,
Imagine Schools National Academic & Character Team

Project #1

Domain: Expressions and Equations

Standard EE.7: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

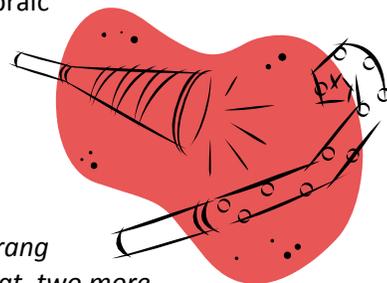
Directions:

1. Read the problem below.

The Murphys love to give parties. Last Friday they gave a party and the doorbell rang 15 times. At the first ring, one guest arrived. Each time the doorbell rang after that, two more guests arrived than the time before.

On Saturday they had another party. At the first ring of the doorbell a single guest arrived, at the second ring two guests appeared, at the third ring three guests and so on. If the doorbell rang 20 times Saturday night, how many guests attended? Was this party bigger than Friday's? How do you know?

2. Draw a picture to show one way to solve this problem.
3. Draw a table to show a second way to solve.
4. Write your answer below and explain how you arrived at your solution.



Project # 2

Domain: Ratios and Proportional Relationships

Standard RP.7: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Directions:

1. Complete the following three problems to apply your understanding of percentages and ratios.

Problem #1:

Al's Awesome Autos advertised a special sale on cars - Dealer cost plus 5%!!
Jack and Margaret bought a luxury sedan for \$23,727.90. What was the dealer's cost?

Problem #2:

You and some friends went out to T.G.I. Fridays for dinner. You ordered a root beer, sweet potato fries and cheese quesadillas. The total bill came to \$21.86. Your dad has told you many times that it's important to leave a good tip; about 20%. You have \$26.00 in your wallet. Can you leave a 20% tip? How much would the total be if you left a 20% tip? Can you cover the cost?

Problem #3:

Builders have observed that residence windows are most attractive if they have the width to length ratio 3:5. If a window is to be 48 inches wide, what should its length be for the most attractive appearance?

2. Create one original problem involving a percentage (discount or tax).
3. Create one original problem involving a ratio or part/whole relationship.
4. Solve both and keep the answer key.
5. Challenge a friend to solve your problems.

Project # 3

Domain: Number System

Standard NS.7: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Directions:

Complete the two problems below.

Problem 1:

Using exactly four 4's and any operations [+ , - , x , ÷ , ()] write an expression to equal each of the following:

$1 = \underline{\hspace{2cm}}$

$4 = \underline{\hspace{2cm}}$

$2 = \underline{\hspace{2cm}}$

$5 = \underline{\hspace{2cm}}$

$3 = \underline{\hspace{2cm}}$

*Example: $16 = \underline{4 \times 4 \times 4 \div 4}$

Problem 2:

Find three different ways to fill in operations in the boxes below to make the equations true.

$6 \square 1 \square 2 \square 2 = 5$

*Hint: Operations include: +, -, x, ÷, ()

$6 \square 1 \square 2 \square 2 = 5$

$6 \square 1 \square 2 \square 2 = 5$

Project #4

Domain: Expressions and Equations

Standard EE.7: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Directions:

Identify a situation in which it would be appropriate to estimate a measurement. Determine the level of accuracy needed and give an explanation for why the situation you have presented is acceptable for estimation. Then, select the appropriate method of estimation.

Example: You need to know approximately how much grass seed to buy to cover your backyard. You calculate the perimeter and area of the yard. Research to find out how much area on bag of grass seed will cover. Then estimate the number of bags of grass seed needed to cover the surface area of the backyard.



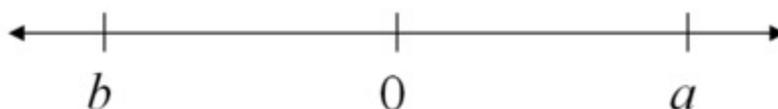
Project #5

Domain: Number Systems

Standard NS.7:1.c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Directions: Solve the first problem, create other problems in part 2.

1. Using what you know about number lines, complete the problem below.



On the number line above, the numbers a and b are the same distance from 0.

What is $a+b$? Explain how you know.

2. Use a picture or physical objects to illustrate and create a problem for others to solve:

- $3 - 7$
- $-3 - 7$
- $-3 - (-7)$
- $(-3)(-7)$
- $21 \div (-3)$

Example for $13 - (-8)$:

At noon on a certain day, the temperature was 13 degrees.

At 10:00 p.m. the same day, the temperature was -8 degrees.

How many degrees did the temperature drop between noon and 10:00 p.m.?

Project #6

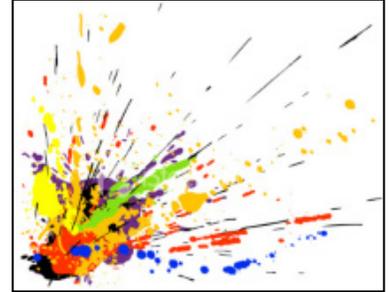
Domain: Ratios and Proportions

Standard RP.7: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Directions: Solve the following problems.

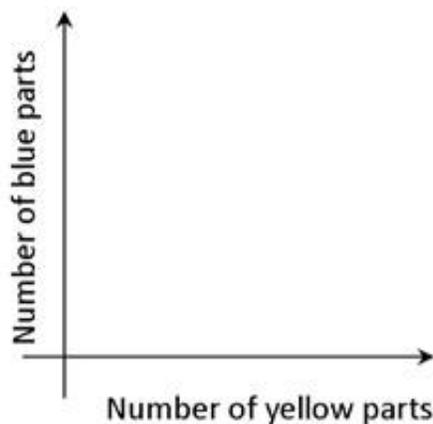
The students in Ms. Baca's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.



	A	B	C	D	E
Yellow	1 part	2 parts	3 parts	4 parts	6 parts
Blue	2 part	3 parts	6 parts	6 parts	9 parts

- How many different shades of paint did the students make?
- Some of the shades of paint were bluer than others. Which mixture(s) were the bluest? Show work or explain how you know.
- Carefully plot a point for each mixture on a coordinate plane like the one that is shown in the figure. (Graph paper might help.)



- Draw a line connecting each point to (0,0). What do the mixtures that are the same shade of green have in common?

Project #7

Domain: Number System

Standard NS.7: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Directions: Read the situation carefully and solve the problem.

Ocean water freezes at about -212°C . Fresh water freezes at 0°C . Antifreeze, a liquid used to cool most car engines, freezes at -64°C . Imagine that the temperature is exactly at the freezing point for ocean water. How many degrees must the temperature drop for the antifreeze to turn to ice?



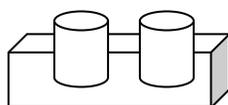
Project #8

Domain: Geometry

Standard G.7: Draw, construct, and describe geometrical figures and describe the relationships between them and solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

1. Construct two different three-dimensional models of your own using a cereal box, soup cans, or other household recyclable items. Each of your two constructions should include at least two different shapes combined. Example: My construction might include two soup cans taped on top of a cereal box.



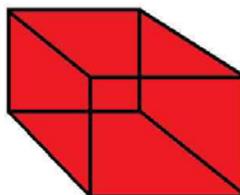
2. Use the formulas and examples below to calculate the surface area and volume for each of your composite shape creations.

Surface Area - The number of square units covering a 2 or 3-D shape.

Example: If you were painting a bedroom and you needed to know how many cans of paint to buy, the surface area would have to be found.

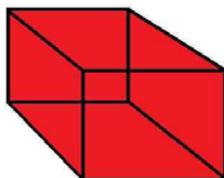
To find the surface area of a cube, use the formula:

$$SA=2(wh+lw+lh)$$



Volume - The number of cubic units that fill a 3-D shape. Volume can also be referred to as capacity.

Example: If you needed to know how much water your bath tub could hold.



To find the volume of a cube, use the formula:

$$V=lwh$$

*Use help on the internet (with supervision) to find formulas for surface area and volume of additional three dimensional shapes.

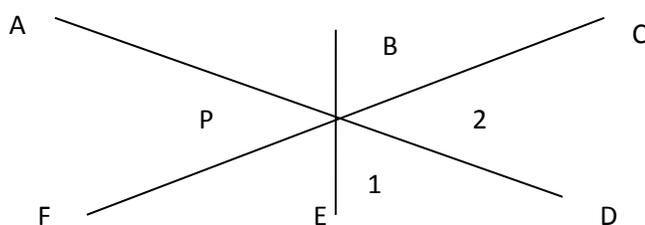
Project #9

Domain: Geometry

Standard G.7: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

1. Study the diagram and coinciding information below.



Angle 1 is vertical with $\angle FPE$. Angle 2 is vertical with $\angle APF$.

In each case these pairs of angles form an X.

$\angle APF$ and $\angle APC$ are supplementary because they form the straight line FC.

$\angle APC$ and $\angle CPD$ are supplementary because they form the straight line AD.

$\angle APB$ and $\angle EPD$ are vertical.

$\angle EPF$ and $\angle EPC$ are supplementary because they form the straight line FC.

2. Find 2-3 real objects in your home or neighborhood that demonstrates one or more of the same relationships expressed in the diagram above.
3. Take pictures of each of the objects you found and either download the pictures and paste them into an electronic document(s) or create a poster and paste your pictures on the poster. *If you do not have access to a digital camera and source for printing pictures, you may draw a picture of your objects instead.
4. Finally label each line, each angle, and each corresponding relationship. Use words to describe the angles and relationships formed by the intersecting lines on your document or poster (as done in the example above).

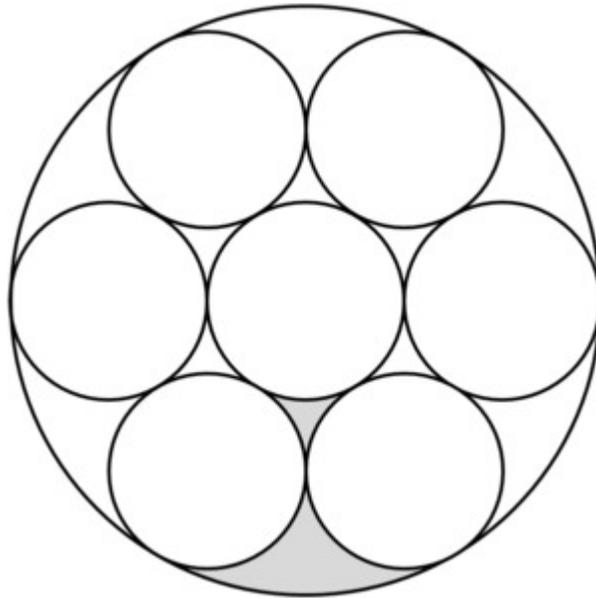
Project # 10

Domain: Geometry

Standard G8: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

The figure below is composed of eight circles. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.



Calculate:

- What is the area of the large circle?
- What is the area of the shaded part of the figure?

Project # 11

Domain: Geometry

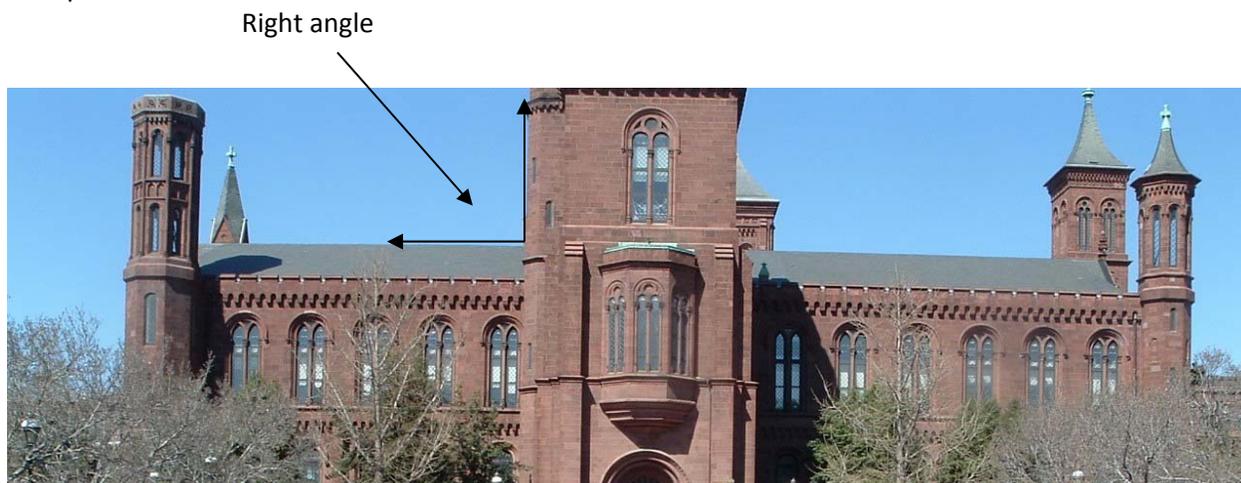
Standard G.7: Draw, construct, and describe geometrical figures and describe the relationships between them.

Directions:

1. Using a sheet on graph paper and a ruler, draw an example of each type of angle: acute, obtuse, right, straight, complementary, supplementary, and vertical.
2. Label each of your angles.
3. Then, find an example of each in the real world (i.e., in your home, neighborhood, or natural environment, at the grocery store, baseball field, in your car, etc.).
4. Take a picture of each example and print it out and paste it on a piece of paper. Label each picture with the correct label.

*If you do not have a camera, draw a sketch of each example you find in the real world and label as using the appropriate geometric terms.

Example:



Project #12

Domain: Geometry

Standard G.7: Draw, construct, and describe geometrical figures and describe the relationships between them and solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

1. Using different colored sheets of construction paper and a ruler, draw 6 congruent triangles.
*Remember, two triangles (polygons) are congruent if all pairs of corresponding sides are congruent, and all pairs of corresponding angles are congruent.

Example of two congruent triangles:

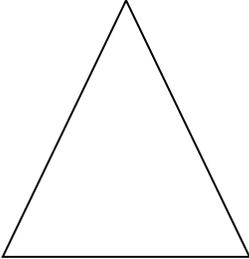
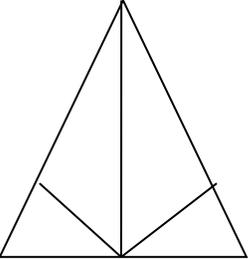
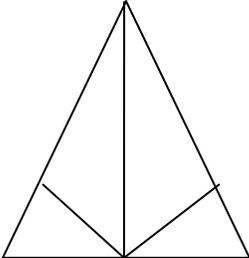
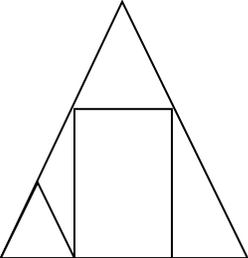
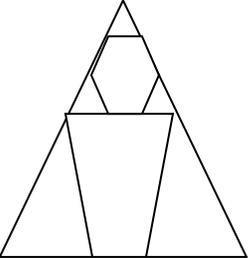
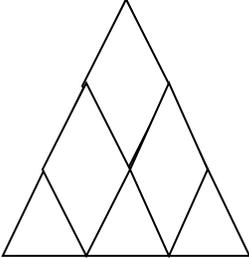


2. Label each triangle with a different number (#1, #2, #3, etc.).
3. Cut each of your numbered congruent triangles out of construction paper.
4. Glue "Triangle #1" on a piece of poster board just as it is. This will serve as your template or model triangle.
5. Next, take triangles #2 & #3. Using your ruler, divide each of them into two sets congruent triangles. Use a compass to make sure your angles and length of sides are congruent on triangle #2 and #3. Label another part of your poster board, "Triangle #2" and "Triangle #3". Paste your new congruent triangles for each under the appropriate label. See the following page for an example.
6. Deconstruct your three remaining triangles (#s 4, 5, & 6), to create a variety of different polygons. You are free to cut them into as many shapes as you want, but should paste them on the poster board to show that they can be reconstructed to form a triangle congruent to the rest of your triangles. Paste your polygons for triangles 4, 5, & 6 on your poster board, labeling each.

*See next page for an example of a completed poster board.

Project #12 Examples

Examples

Triangle #1	Triangle #2	Triangle #3
		
Triangle #4	Triangle #5	Triangle #6
		

Project #13

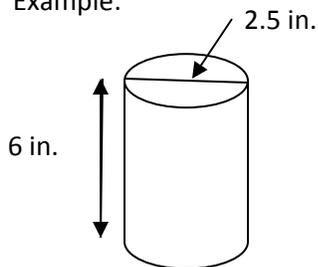
Domain: Geometry

Standard G.7: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

1. Get a tape measure or ruler, or make your own tape measure using paper taped together with accurate measurements denoted on the paper created using a ruler. *Use inches for the following problems.
2. Find five examples of cylinders in your home, grocery store or neighborhood.
3. Take a picture with a digital camera or draw a picture on notebook paper or poster board of each of the five cylinders you have found.
4. Measure the volume of each of the cylinders you found. Show the base and height measures for each cylinder. Show all of the calculations you completed to solve to find the volume (below your picture of each cylinder).

Example:



Base = 2.5 in.

Height = 6 in.

Volume: Area of base x height = $(3.14 \times 2.5)^2 \times 6 =$ in.

$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ \text{Area of circle} &= 3.14 \times (\text{radius})^2 \\ \text{Area of base} &= (3.14 \times 2.5 \text{ in.})^2 \end{aligned}$

Project #14

Domain: Geometry

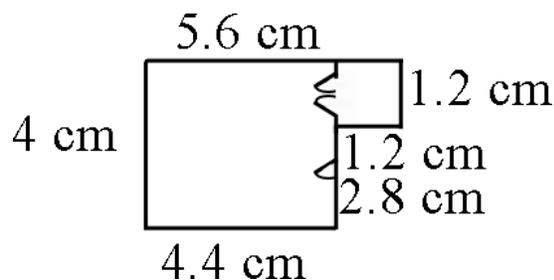
Standard G.7: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Directions:

Part 1:

1. Read the problem below then study the scale drawing.

Kiera showed her best friend a scale drawing of a new game room her father is going to build for her and her brothers and sisters.



2. Solve: If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Kiera's room?
3. How much carpet needs to be purchased to cover the entire floor?

Part 2:

1. Design your own game room using Kiera's scale drawing as an example.
2. Using graph paper, determine the scale and be sure to write it in the top right corner of your paper.
Example: (1 cm = 3 ft).
3. Then, create a scale drawing of the game room you have designed. Include game areas, furniture, and more to give the best visual representation possible.
4. Calculate the area of the floor that is showing versus the area of the floor under the furniture.

*Be sure to use a ruler to create straight lines and accurate measures.

Project # 15

Domain: Statistics & Probability

Standard SP7: Investigate chance processes and develop, use, and evaluate probability models.

SP.7.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.



Directions:

Read the following problem:

Members of the seventh grade math group have nominated a member of their group to be class president. Every student in seventh grade will cast a vote. There are 2 candidates in the race and a candidate needs at least 50% of the votes to be elected. The math group wants to conduct an opinion poll to assess their candidate's prospects. There are almost 500 students in the seventh grade at their school. They do not have the resources to interview all seventh graders so they have decided to interview a sample of 40 seventh graders. They will obtain the seventh grade list of names from their school principal's office and select the sample from this list. They plan to ask each student in the sample whether they plan to vote for their candidate or the other candidate.

Random Number Table Generated in Excel

196	14	57	441	219
459	284	356	306	119
358	241	406	122	390
238	98	392	433	256
335	189	24	260	452
468	106	28	294	46
20	385	37	109	4
437	70	464	471	432
454	474	1	280	117
492	390	154	115	336
460	377	101	312	350
115	126	64	333	291
445	297	449	171	234
438	224	357	13	500
288	284	254	86	173
449	340	11	9	387
359	133	494	31	458
217	174	343	3	350
171	195	127	141	276
299	246	394	164	294

- How should the students select the sample of 40 to have the best chance of predicting the outcome of the vote? Describe clearly how they could use the random number table provided below to select the sample of 40 students. "Clearly" means that someone other than you could duplicate the sampling process by following your description.
- Suppose that all 40 students selected from the list of seventh graders in the school respond to the survey, and the results showed that 18 students would vote for the math group's candidate. In order to get elected, a candidate must receive at least 50% of the votes. Some members of the

math group believe that on the basis of this sample survey outcome it is unreasonable to think that their candidate can win. Others in the group believe they should take another sample to check their initial results. Based on the initial survey results, should the math group students be discouraged, or is it reasonable to think their candidate might win? Justify your response statistically.

Project # 16

Domain: Statistics & Probability

Standard SP.7: Use random sampling to draw inferences about a population.

Directions:

1. Look at the following data set. It represents the height in centimeters of a group of students:

Student	Height in cm
Tim	145
Liz	136
Latoya	154
Paloma	178
Sean	173
Juan	164
Andrea	144
Allie	170
Tyrone	183
Juanita	144

1. Answer the following questions based on the data set above.

What is the mode of the set? _____

What is the range of the set? _____

Whose height is closest to the median height for the set? _____

Whose height is closest to the mean height for the set? _____

2. Create a scatter plot and box and whisker plot using all of the above data.

*If you need help, search on the internet to find examples of scatter plots and box and whisker plots. Be sure to do so only with permission from your parent or guardian.

Project #17

Domain: Statistics and Probability

Standard SP.7: Use random sampling to draw inferences about a population.



Directions:

1. This is an open-ended data project in which you get to be the designer. Your job is to design an experiment based on a question you would like to find the answer to by collecting responses and data.
2. Start with your question, then come up with a hypothesis based on what seems reasonable. *Your hypothesis should reflect your background understanding and knowledge (and any research you have done related to the question you are trying to answer.
3. Develop an organized means of collecting data.
4. Evaluate, analyze and display your data.
5. Then formulate a conclusion.

*Include each of the above steps in a short written report. Be sure to include your hypothesis, background research, rationale for the experiment and any tables, graphs and charts you used to collect, organize or display your data, results and analysis.

Project #18

Domain: Expressions and Equations

Standard EE.7: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

*You have tried many ways to solve problems throughout this Math Challenge Packet. Already you know that when one strategy does not lead you to a solution, you back up and try something else. Sometimes you can find a smaller problem inside the larger one that must be solved first. Sometimes you need to think about the information that is missing rather than what is there. Sometimes you need to read the problem again and look for a different point of view. Sometimes you need to tell your brain to try to think about the problem in an entirely different way – perhaps a way you have never used before. Looking for different ways to solve problems is like brainstorming. Try to solve this problem. You may need to **change your point of view**.*

Directions:

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

1. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.
2. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

Extensions

The following projects are based on standards you will learn in eighth grade. They should be challenging for you at this point. Try them out and save your work along the way.

Project #19

Strand: Number System

Standard NS.8: Work with radicals and integer exponents.

Directions:

1. Write out the following in words. For example, 10^{26} would be ten to the twenty-sixth power.

10^{26}	-2,900	8.616	8×10^3	$\frac{3}{5}$
7:3	$23\frac{1}{8}$	85%	0.006	7,000,000,000

2. Solve the problem below.

The evil ruler, Gonglo has locked the beautiful, intelligent Princess Alexa in a loft high up in the castle. To be released, she must find the first perfect square number greater than 100 whose digits do not sum to a perfect square! Can you help her find the number?

3. Create a challenge problem of your own using the formula for finding a perfect square.

Project #20

Domain: Functions

Standard F.8: Use functions to model relationships between quantities.



Directions:

1. Use the information below to answer the riddle.

Amy, Connor, Jalia, Stella and Gonzo live in apartments A, B, C, D, and E in the same building. Their ages are 9, 10, 11, 13, and 14. They each get up at a different time each morning, either 6:00, 7:00, 7:30, 8:00, or 8:15. Using the clues below, determine the apartment, age and rising time for each student.

- Connor gets up at 6:00 and is older than the student in Apt. B.
- Amy is 11 and gets up two hours later than the boy in Apt. A.
- The ten-year old girl lives in Apt. B and it isn't Stella.
- The youngest student, a girl, doesn't live in Apt. E.
- The person in Apt. B gets up later than everyone else.
- Gonzo is younger than Connor and he lives in Apt. D.
- The shades go up at 7:30 in Apt. C.

Time	Apartment	Age
Amy		
Connor		
Jalia		
Stella		

2. Create your own riddle for a friend to solve.