# Imagine Schools
# Summer Math Challenge

## Grade Eight
## Answer Key

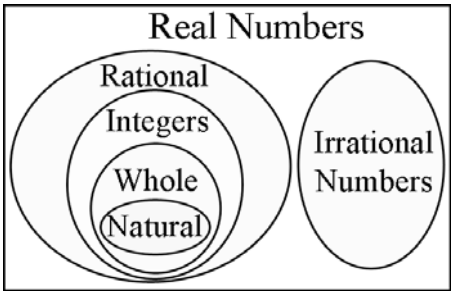
When completing the problems we need to show all of our work and show all of our thinking. In this answer key, important information that was used to solve the problem is bolded and underlined to show our thinking. We also provided speech balloons to show our thoughts and help you see how we thought through the problem when we were solving it. Compare your work to ours, especially if your answer is different than our answer.

# Project #1

**Domain:** Number System

**Standard NS8:** Know that there are numbers that are not rational, and approximate them by rational numbers.

**Directions:** Pretend your 10-year-old younger brother or sister knows nothing about integers, rational, whole, natural, and irrational numbers. Write two to three paragraphs explaining the graph below. Be sure to use language that a fourth or fifth grader can understand.




| Vocabulary | Definitions |
|---|---|
| Real Numbers | All the numbers that can be represented by points on a number line. |
| Rational | All numbers that can be written as a ratio of two integers. (15/16 and 37) |
| Integers | The set of counting numbers, their opposites, and zero; (-2, -1, 0, 1, 2….)<br>NOT 15/8, -0.98 |
| Whole | The members of a set (1,2,3,4)<br>NOT -3, 0.56, 100 ¾ |
| Natural | Or counting numbers, numbers that are used to count (1,2,3,4…) |
| Irrational | Numbers that cannot be expressed as a ratio of two integers. (π) |

**Solution:**

Answers will vary (but not too much)

**Rubric:**

+ Answer should be in a paragraph form with an introduction of this problem, explanation and definition of terms and circular illustration and chart and then a conclusion. Students could use the metacognition frame below:

+ Must use the vocabulary correctly.

I know that I know something about __real numbers.__

First, I know_____

_____. Second, I know _____. (continue with body of the paragraph).

Now you know what I know about real numbers.

# Project # 2

**Domain:** Number System

**Standard NS8:** Know that there are numbers that are not rational, and approximate them by rational numbers.

**Directions:** Compare each pair of numbers using <, >, or =. Write one to two sentences after each problem explaining your answer.

**Solution:**

- $-\frac{11}{21}$ [ > ] $-\frac{13}{21}$

**Explanation:** If you visualize a number line, the negative number that is closer to 0 is the larger number. So -11/21 is the larger number. The symbol shows the open side toward the larger number and the point toward the smaller number.

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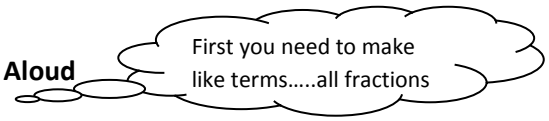
- $-\frac{7}{5}$ □ -1.35

**Think Aloud**




**Explanation:**

1) First, you need to convert the numbers into like terms. (I took off the negative, but will make sure I put it back on after I finish these calculations.)
   1.35 = 1 35/100, when reduced that is 1 7/20.
   1 7/20 = 27/20
   7/5 = 28/20
2) Now you compare the two (both are negative, so on the left side of 0 on a number line)
   - 28/20 [ > ] - 27/20

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**Solution:**

- $2\frac{3}{4}$ □ 2.75

**Explanation:**

1) First, you need to convert the numbers into like terms.
   2.75 = 2 75/100 when reduced that is 2 3/4
2) $2\frac{3}{4}$ [ = ] $2\frac{3}{4}$

Now, create two to three similar problems and challenge an adult or friend to solve them.

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**Solution:**

Answers will vary

**Rubric:**

+ One should have a negative numbers
+ All should be working with fractions and decimals

# Project #3

**Domain:** Number System
**Standard NS8:** Know that there are numbers that are not rational, and approximate them by rational numbers.

**Directions:** List the numbers $\frac{2}{3}$, $-\frac{2}{3}$, 1.2, $\frac{4}{3}$, $-\frac{4}{3}$, -1.2, and $-\frac{7}{4}$ from least to greatest and then locate the numbers on the number line. You will label the number line below to fit your purposes.

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**Solution:**

**Think Aloud**

First you need to make all numbers have like terms and give all fractions with like denominators.

**Explanation:**

1) Make conversions so all the numbers are fractions or improper fractions (no mixed numbers)
   - ✓ 1.2 = 1 2/10 = 1 1/5 = 6/5
   - ✓ -1.2 - -6/5
2) Now make them all with common denominators
   - ✓ 2/3 = 40/60 and -2/3 = -40/60
   - ✓ 6/5 = 72/60 and -6/5 = -72/60
   - ✓ 4/3 = 80/60 and -4/3 = -80/60
   - ✓ -7/4 = -105/60
3) Now order them around 0 (with the positive going from small to large to the right of 0 and the negative going from small to large to the left of )

| -7/4 | -4/3 | -6/5 | -2/3 | 0 | 2/3 | 6/5 | 4/3 |
|---|---|---|---|---|---|---|---|
| -105/60 | -80/60 | -72/60 | -40/60 | 0 | 40/60 | 72/60 | 80/60 |

# Project #4

**Domain:** Number System

**Standard NS8:** Know that there are numbers that are not rational, and approximate them by rational numbers.

**Directions:**

Without using the square root button on your calculator, estimate $\sqrt{800}$ as accurately as possible to 2 decimal places. (It is worth noting that $20^2=400$ and $30^2=900$.)

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**Solution:**

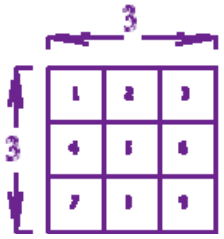
**Review:**

(Hint: Review the meaning of Square and Square Root.)

**How to Square A Number**

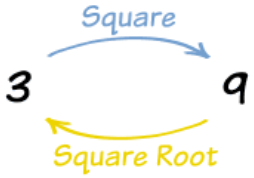
To square a number, just multiply it by itself ...

**Example: What is 3 squared? 3 x 3 = 9 or $3^2$**




**Square Roots**

A **square root** goes the other way:




3 squared is 9, so a **square root of 9 is 3**

A square root of a number is ... a value that can be **multiplied by itself** to give the original number.

A square root of **9** is ... **3**, because **when 3 is multiplied by itself** you get **9**.

It is like asking: What can I multiply by itself to get this?


**To help you remember** think of the root of a tree:

*"I know the tree, but what is the root that produced it?*"

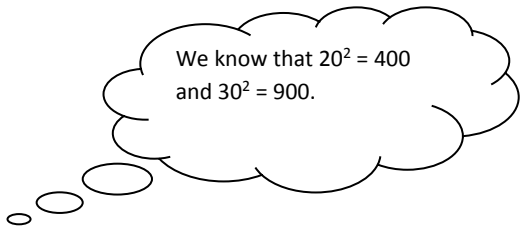
In this case the tree is "9", and the root is "3".

**Solution:** Using the definition of a square root

We know that:

$20^2=400$ and $30^2=900$ so $20 < \sqrt{800} < 30$

Choosing successive approximations carefully, we see that:




**Think Aloud**

| $n$ | $n^2$ | $m^2$ | $m$ |
|---|---|---|---|
| 28 | 784 | 851 | 29 |
| 28.2 | 795.24 | 800.89 | 29.3 |
| 28.28 | 799.7584 | 800.3241 | 28.29 |
| 28.284 | 799.984656 | 800.041225 | 28.285 |

So $\sqrt{800} \approx 28.28$

# Project #5

**Domain: Functions**
**Standard F8:** Use functions to model relationships between quantities.

**Directions:**
Solve the problem below.

Linda traveled **110 miles** in **2 hours**. If her speed remains constant, how many miles can she expect to travel in **4.5 hours**? Answer the question in complete sentences and show your work.

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**Solution:** Computation Problem, so use the information given.

Step 1: 110 ÷ 2 = 55mph
Step 2: 55 x 4.5 mph = 247.50 miles

Then, create a problem of your own involving linear functions and constant rates of speed over a specified distance.

**Solution:**
Answers will vary
**Rubric:**

+ Must involve functions
+ Must involve constant rates of speed over distance

# Project # 6

**Domain:** Functions
**Standard F8:** Define, evaluate, and compare functions.

The table shows a relation between x and y.

| x | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| y | 7 | 10 | 13 | 16 |

Which of these equations expresses this relation? Explain your answer in complete sentences.

A. $y = x + 5$
B. $y = x \pm 5$
C. $y = \frac{1}{3}(x \pm 1)$
D. $y = 3x + 1$

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**Solution:** Guess and Check Strategy

Step 1: You can insert the information in each equation to see which one expresses the required relationship

A. Does $y = x + 5$? No, $3 + 5 \neq 10$
B. Does $y = x \pm 5$? No, it does not work.
C. Does $y = 1/3 (x \pm 1)$ No, it does not work.
D. Does $y = 3x + 1$? Yes, it works – see below.
   $3 (2) + 1 = 7$
   $3 (3) + 1 = 9$
   $4 (3) + 1 = 13$
   $5 (3) + 1 = 16$

How might you use information like the above in the real world? Describe a situation in which you would need to create a table and apply your understanding of a function.

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**Solution:**
Answers will vary
**Rubric:**
+ Must be a real world example
+ Create a table

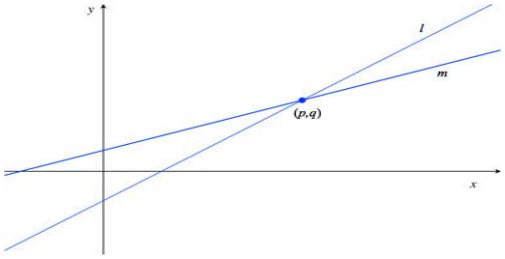
# Project #7

**Domain:** Expressions and Equations

**Standard EE8:** Understand the connections between proportional relationships, lines, and linear equations.

**Directions:**

The figure below shows the lines $l$ and $m$ described by the equations $4x-y=a$ and $y=2x+b$, for some constants $a$ and $b$. They intersect at the point $(p, q)$.




**Solution:**

Review Slope:

The slope of a line is the ratio of the change in y over the change in x.

Simply put, find the difference of both the y and x coordinates and place them in a ratio.

This is the formula for finding the slope of a line.

*m is the variable used for the slope.*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

1. **How can you interpret a and b in terms of the graphs of the equations above?"**

**Solution:**

1. If we put the equation $4x-y=a$ in the form $y=4x-a$, we see that the graph has slope 4. The slope of the graph of $y=2x+b$ is 2. So the steeper line, $l$, is the one with equation $y=4x-a$, and therefore $-a$ is the y -coordinate of the point where l intersects the y -axis. The other line, m, is the one with equation $y=2x+b$, so b is the y -coordinate of the point where m intersects the y -axis.

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2. **Imagine you place the tip of your pencil at point (p, q) and trace line $l$ out to the point with x -coordinate p + 2. Imagine I do the same on line m. How much greater would the y -coordinate of your ending point be than mine?**

**Solution:**

2. The line $l$ has slope 4. So on $l$, each increase of one unit in the x -value produces an increase of 4 units in the y -value. Thus an increase of 2 units in the x -value produce an increase of $2 \cdot 4 = 8$ units in the y -value. The line m has slope 2. So on $L_2$, each increase of 1 unit in the x -value produces an increase of 2 units in the y -value. Thus an increase of 2 units in the x -value produces an increase of $2 \cdot 2 = 4$ units in the y -value.

Thus your y -value would be $8-4=4$ units larger than my y -value.

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**Now, create a function of your own. State the coordinates of two points along the line. Challenge a friend or a parent to find the slope.**

**Solution:**

Answer will vary

**Rubric:**

+ Must be a function with the coordinates of two points along the line.
+ Must ask for the slope

# Project #8

**Domain:** Functions
**Standard F8:** Use functions to model relationships between quantities.

Which of the following could be modeled by $y=2x+5$? Answer YES or NO for each one.

a. There are initially 5 rabbits on the farm. Each month thereafter the number of rabbits is 2 times the number in the month before. How many rabbits are there after $x$ months? YES NO
b. Joaquin earns $2.00 for each magazine sale. Each time he sells a magazine he also gets a five-dollar tip. How much money will he earn after selling $x$ magazines? YES NO
c. Sandy charges $2.00 an hour for babysitting. Parents are charged $5.00 if they arrive home later than scheduled. Assuming the parents arrived late, how much money does she earn for $x$ hours? YES NO
d. I have a sequence of integers. The first term of the sequence is 7 and the difference between any consecutive terms is always equal to 2. YES NO
e. Sneak Preview is a members-only video store. There is a $2.00 initiation fee and a $5.00 per video rental fee. How much would John owe on his first visit if he becomes a member and rents $x$ videos? YES NO
f. Andy is saving money for a new CD player. He began saving with a $5.00 gift and will continue to save $2.00 each week. How much money will he have saved at the end of $x$ weeks? YES NO

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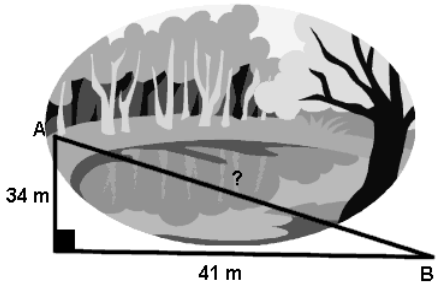
**Solution:**

a. **NO:** If $y$ is the number of rabbits at time $x$ months, then in this scenario the number of rabbits starts with 5 and doubles every months. This situation is modeled by the function $y=5\cdot 2x$.
b. **NO:** If $y$ is the amount of money in dollars Joaquin earns for selling $x$ magazines, then for each magazine sold, Joaquin actually gets 2+5=7 dollars. So this situation is modeled by the function $y=2x+5x=7x$.
c. **YES:** If $y$ is the amount of money Sandy earns for $x$ hours of babysitting then $y=2x+5$ models this situation. She earns 2 dollars per hour and the extra term of 5 represents the 5 dollar penalty Sandy charges parents for coming home late.
d. **YES:** In this case we can think of $y$ as the $x^{th}$ term of the sequence. So $7=2(1)+5$, $9=2(2)+5$, $11=2(3)+5$,…. In general we have: $y=2x+5$
e. **NO:** If $y$ is the amount of money in dollars John owes for renting $x$ videos, then this situation is modeled by the function $y=5x+2$. John pays 5 dollars per video and a onetime initiation fee of 2 dollars.
f. **Yes:** If $y$ is the amount of money (in dollars) Andy has saved after $x$ weeks then this situation is modeled by the function $y=5+2x$. Andy already has 5 dollars to begin with and he saves an additional 2 dollars per week.

# Project # 9

**Strand:** Geometry
**Standard G8:** Understand and apply the Pythagorean Theorem.




The Pythagorean Theorem is $\mathbf{a^2 + b^2 = c^2}$. C is the side opposite the right angle. In this particular problem "C" is the unknown side.

**Pythagorean Theorem in the Real-World**

To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond? Choose: 22, 34, 53, or 75. Show all of your work and explain how you came up with your solution.

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**Solution**

**Part I:** Calculate the distance around the pond

34 + 41 = 75 meter

Use the formula; insert the numbers that you know.

**Part II:** Calculate the distance through the pond by using the Pythagorean Theorem ($a^2 + b^2 = c^2$)

$34^2 + 41^2 = c^2$

$1156 + 1681 = 2837$

$2837 = c^2$

$C = \sqrt{2837}$

$C = 53.26$ (53 nearest meter)

**Part II:** Distance saved

75 – 53 = 22

**Part IV:** Choose from 22, 34, 53, or 75

**Answer:** 22 meters

53 = the distance through the pond. If you picked this – you didn't read through the whole problem. There is another step.
75 = the distance around the pond. Again you didn't read the whole question carefully.
34 = I hope you didn't pick this one because it was given as the distance of one side of the walk. Which means you didn't try to work this out.

**Think Aloud**

# Project #10

**Domain:** Number System
**Standard NS8:** Know that there are numbers that are not rational, and approximate them by rational numbers.
**Imagine Subject Area:** Financial Literacy

**Directions:** The United States owes approximately $14,300,000,000,000.00 in National debt. There are approximately 300 million people in the United States. Assume the United States would like to pay off the debt in one lump sum. How much would each American have to pay to pay off the National debt?

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**Solution:**
Divide $14,000,000,000,000.00 by 300,000,000 = $47,666.67

If each American paid $2,000 a year towards the National debt how many years would it take for the country to pay off what it owes? (Assume that there is no interest on the debt.)

**Solution:**
Divide $47,666.67 by $2,000 = 23.8333, now round off to 24 years

# Project # 11

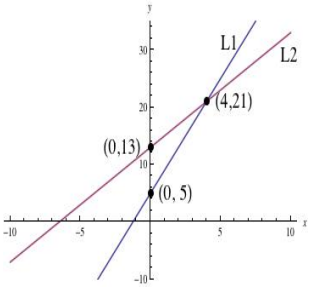
**Domain:** Equations and Expressions

**Standard EE8:** Analyze and solve linear equations and pairs of simultaneous linear equations

**Directions:**

Consider the graph below showing two lines, L1 and L2.

1. Find the two corresponding linear equations.




**Solution:** Writing a System of Equations

1. The graph shows two lines, L1 and L2 . In slope-intercept form, the equations are of the form y=mx+b. The graph shows the y -intercepts of both lines. Since the y -intercept corresponds to the point where x=0, the y -intercept of a line in slope-intercept form is equal to the value of b. Therefore, we have:

   $L1: y = m_1 x+5$

   $L2: y = m_2 x+13$

   From the graph we can also see that the point (4, 21) lies on both lines. Therefore, x=4, y=21 is a solution to both equations. Substituting the solution into the equations, we have

   $21 = m_1 4+5$

   $21\ m_2 4+13$

   Solving the equations for $m_1$ and $m_2$ we find $m_1 = 4$ and $m_2 = 2$, and so our two equations are

   $y = 4x+5$

   $y = 2x+13$

---

2. Find points other than the ones given in the graph; one that lies on L1 but not on L2 and one that lies on L2 but not on L1 .

**Solution:**

2. We know that a point of intersection of two graphs corresponds to the solution of the corresponding system of equations. These two lines intersect at (4, 21). As lines can intersect at most once, we also know that (4, 21) is the only solution to this system of equations. Therefore, to find a point on one line that is not on the other, we can take any x -coordinate other than 4 (or 0 to avoid the y -intercepts) and solve for the corresponding y -coordinate. For y = 5+4x let us take x = 2. Then we find the corresponding y -value through substitution:

   $y = 5 + 4(2) = 5 + 8 = 13$

   and so (2,13) is a point on L1. We can verify that this point does not also lie on L2, again through substitution

   $y = 13 + 2(2)=13 + 4 = 17$

   We found that at x = 2 the y -coordinate for L2 is not y = 13 but rather y=17. So, we have, in fact, found two points, (2, 13) and (2, 17), which lie on one line but not the other.

3. Name three examples of congruent angles in nature, at school or in the gymnasium.

**Solution:**

Answers will vary

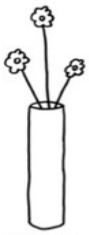
**Rubric:**

+ Must have three examples
+ Must show congruent angles
+ Must use real world examples

# Project #12

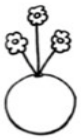
**Domain:** Geometry
**Standard G8**: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

My sister's birthday is in a few weeks and I would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and I want to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.



**Cylinder Vase**
Show off your flowers in this beautiful vase.
10cm X 40 cm
$9.95
4KE09

**Cone Vase**
This vase holds your flowers in place!
16cm X 45cm
$9.95
4KE08



**Sphere Vase**
Doesn't get any more symmetric than this!
18cm X 18cm
$9.95
4KE07

1. Which vase should I purchase?

**Solution:**

1. You should purchase the cylinder vase. If ***r*** is the radius and ***h*** is the height, then, using the fact that the radius is half the diameter, we get

   Cylinder Volume $=\pi r^2 h$
   $=\pi(5)^2 \ (40)\text{cm}^3$
   **$=1000\ \pi\ \text{cm}^3$**

   Cone Volume $=1/3\ \pi r^2 h$
   $=1/3\ \pi(8)^2\ (45)\text{cm}^3$
   **$=960\ \pi\ \text{cm}^3$**

   Sphere Volume $=4/3\ \pi r^3$
   $=4/3\ \pi(9)^3\ \text{cm}^3$
   **$=972\ \pi\ \text{cm}^3$**

**Pencil Vase**
This perfect gift for your math teacher!
12cm X 42cm
$9.95
4KE06

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2. How much more water does the largest vase hold than the smallest vase?

**Solution:**

Cylinder Volume$-$Cone Volume$=1000\pi\ \text{cm}^3-960\pi\ \text{cm}^3=40\pi\ \text{cm}^3$

---

3. Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?

**Solution:**

1. If the diameter decreases by 2 cm, then the radius decreases by 1 cm. Now the cone holds more water:

   Cylinder Volume$=\pi r^2h$
   $=\pi(4)^2(40)\ cm^3$
   $=640\pi\ cm^3$

   Cone Volume $=1/3\pi r^2h$
   $=1/3\pi(7)^2(45)\ cm^3$
   $=735\pi\ cm^3$

   Sphere volume $=4/3\pi r^3$
   $=4/3\pi(8)^3 cm^3$
   $=682\ 2/3\ \pi\ cm^3$

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4. The vase company designs a new vase that is shaped like a cylinder on bottom and a cone on top. The catalog states that the width is 12 cm and the total height is 42 cm. What would the height of the cylinder part have to be in order for the total volume to be $1224\pi\ cm^3$?

**Solution:**

4. The total volume is the volume of the cylinder plus the volume of the cone. If the cylinder has height x cm then the cone has height $42-x$ cm, since the total height is 42 cm. So the volume of the cylinder plus the volume of the cone is

   $\pi r^2x+1/3\pi r^2(42-x)$ $=\pi(6)^2(x)+1/3\pi(6)^2(42-x)$
   $=36\pi x+12\pi(42-x)$
   $=36\pi x+504\pi-12\pi x$
   $=24\pi x+504\pi$

So to find x we must solve the equation

$1224\pi = 24\pi x+504\pi$
$720\pi x = 24\pi$
$\times = 30cm$

---

5. Design your own vase with composite shapes, determine the volume, and write an ad for the catalog.

**Solution:**

Answers will vary

**Rubric:**

+ Must have composite shapes
+ Must find the volume
+ Ad must give dimensions and use persuasive language

# Project #13

**Domain:** Geometry
**Standard G8:** Understand congruence and similarity using physical models, transparencies, or geometry software.

Research and compare the speed, height, track length, and duration of ride for two roller coasters. Choose two pictures of roller coasters from the web site www.joyrides.com (Vortex and King Cobra, for example). Then do research online to find the answers to the questions below. Be sure to include the proper units after each measurement.

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**Solution**

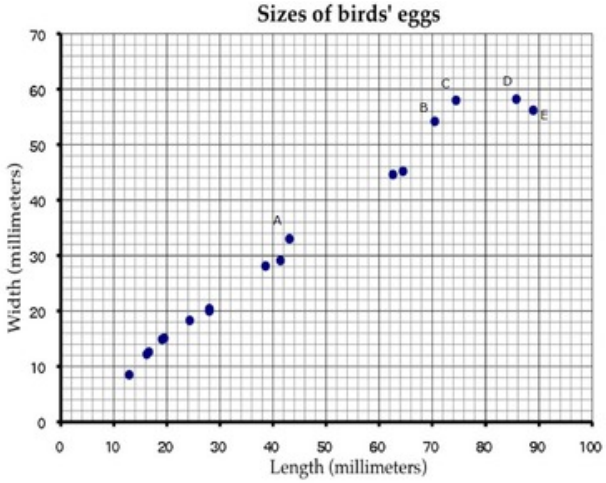
We picked *Raven* and *Voyager* for our examples.

| **Coasters** | | |
|---|---|---|
| | **Name of Coaster:** ***Raven*** | **Name of Coaster:** ***Voyage*** |
| **Speed** | 50 mph | 67.4 mph |
| **Height** | 110 ft | 173 ft |
| **Track Length** | 2800 ft | 6422 ft |
| **Duration** | 90 seconds | 165 seconds |

# Project #14

**Domain:** Statistics and Probability
**Standard:** Investigate patterns of association in bivariate data.

**Directions:**
This scatter diagram shows the lengths and widths of the eggs of some American birds.

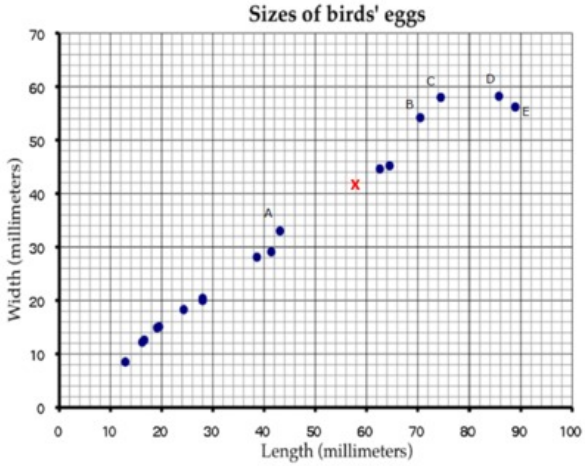



See Questions below:

a. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.

**Solution:**

a. The graph shows the x. You find this by going across the x axis (length) to 57.8 and then go up the y axis (width)



b. What does the graph show about the relationship between the lengths of birds' eggs and their widths?

**Solution:**

b. There seems to be a positive linear relationship between the length and width of the eggs.

---

c. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?

**Solution:**

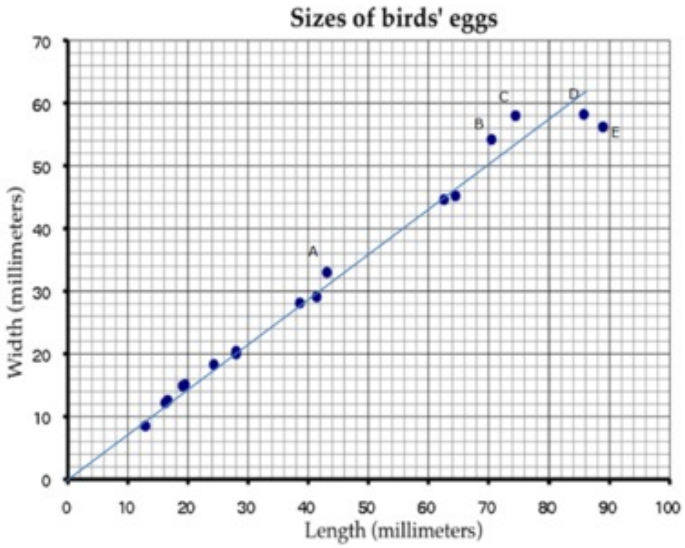
c. The line below appears to fit the data fairly well. Since it passes through (0, 0) and (50, 36), it's slope is $\frac{36}{50} = 0.72$, so the equation of the line is $y=0.72x$. If $x = 35$, then our line would predict that $y = 0.72 \cdot 35 = 25.2$. So we would expect the width of these eggs to be, on average, about 25 mm. Answers using different lines can vary up to 1 mm in either direction.

---

d. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.

**Solution:**

d. Without reading off precise numerical values from the plot, we can see that eggs *C* and *D* have very nearly the same width, but egg *D* is about 12 millimeters longer than egg *C*.



e. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

**Solution:**

e. First we note that egg *E* certainly has a higher length-to-width ratio than *C* or *D*, since it is both longer and narrowed. Similarly, *E* has a higher ratio than *B* because it is significantly longer, and only a tad wider. It is harder to visually identify the difference between *A* and *E*, we compute their respective length-to-width ratios numerically, which turn out to be approximately 1.3 for *A* and 1.6 for *E*. So *E* has the greatest ratio of length to width.

# Project #15

**Domain:** Expressions and Equations
**Standard EE8:** Understand the connections between proportional relationships, lines, and linear equations.

**Directions:**
Read the problems carefully and solve each problem a-e

a. DVDs can be made in a factory in New Mexico at the rate of 20 DVDs per $3, but the factory costs $80,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?
b. DVDs can be made in a factory in Colorado at the rate of 10 DVDs per $1.50, but the factory costs $100,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?
c. How much can a buyer save on a million DVDs by buying DVDs from New Mexico instead of DVDs from Colorado?
d. Find an equation for the cost of making *x* number of DVDs in the factory in New Mexico.
e. Find an equation for the cost of making *x* number of DVDs in the factory in Colorado.

---

**Solution I:** Working with unit rates
Look at the information that you know: If DVDs didn't need a factory, the DVDs from New Mexico would just be 20 DVDs per $3. The cost per DVD would be $.15 per DVD, or 15 cents per DVD. However, we have to factor in the factory cost.

The DVDs from Colorado, without the factory, would be 10 DVDs per $1.50, so the unit cost would also be $.15 per DVD, or 15 cents per DVD, the same as in the factory in New Mexico! Consequently, the only difference in the cost of DVDs between New Mexico and Colorado is the cost of the factory, which costs $20,000 more in Colorado, no matter how many DVDs are involved.

---

a. DVDs can be made in a factory in New Mexico at the rate of 20 DVDs per $3, but the factory costs $80,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?

**Solution:**

a. The cost of 1 million DVDs from New Mexico is:

$$80{,}000 \text{ dollars} + 1{,}000{,}000 \text{ DVDs} \bullet \frac{3 \text{ dollars}}{20 \text{ DVDs}}$$

$$= 80{,}000 \text{ dollars} + 50{,}000 \times 3 \text{ dollars}$$

$$= 80{,}000 \text{ dollars} + 150{,}000 \text{ dollars} = 230{,}000 \text{ dollars}$$

The unit cost per DVD is:

$$\frac{230{,}000 \text{ dollars}}{1{,}000{,}000 \text{ DVDs}} = .23 \frac{\text{dollars}}{\text{DVD}} = .23 \frac{\text{dollars}}{\text{DVD}} \bullet \frac{100 \text{ cents}}{\text{dollar}} = 23 \frac{\text{cents}}{\text{DVD}}$$

**So the unit cost is 23 cents per DVD**

b. DVDs can be made in a factory in Colorado at the rate of 10 DVDs per $1.50, but the factory costs $100,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?

**Solution:**

b. The cost of 1 million DVDs from Colorado is:

$$100{,}000 \text{ dollars} + 1{,}000{,}000 \text{ DVDs} \bullet \frac{1.5 \text{ dollars}}{10 \text{ DVDs}}$$

$$= 100{,}000 \text{ dollars} + 100{,}000 \bullet 1.5 \text{ dollars}$$

$$= 100{,}000 \text{ dollars} + 150{,}000 \text{ dollars} = 250{,}000 \text{ dollars}$$

The unit cost per DVD is:

$$\frac{250{,}000 \text{ dollars}}{1{,}000{,}000 \text{ DVDs}} = .25 \frac{\text{dollars}}{\text{DVDs}} = .25 \frac{\text{dollars}}{\text{DVDs}} \bullet \frac{100 \text{ cents}}{\text{dollar}} = 25 \frac{\text{cents}}{\text{DVD}}$$

**So the unit cost per DVD is 25 cents per DVD**

---

c. How much can a buyer save on a million DVDs by buying DVDs from New Mexico instead of DVDs from Colorado?

**Solution:**

c. The difference is 2 cents per DVD. For a million DVDs, the savings would be:

$$1{,}000{,}000 \text{ DVDs} \bullet \frac{2 \text{ cents}}{\text{DVD}} \bullet \frac{\text{dollar}}{100 \text{ cents}} = 20{,}000 \text{ dollars}$$

Of course, the rate of 10 DVDs per 1.5 dollars is the same as 20 DVDs per 3 dollars, so the only difference is the startup cost difference, $20,000, no matter how many DVDs are made, 0 on up! We have also already computed the cost of a million DVDs from each state, so all we really had to do was subtract the cost of the DVDs from Colorado from the cost of the DVDs from New Mexico,

$$250{,}000 \text{ dollars} - 230{,}000 \text{ dollars} = 20{,}000 \text{ dollars}$$

---

d. Find an equation for the cost of making $x$ number of DVDs in the factory in New Mexico.

**Solution:**

d. The cost $C$ of making $\times$ DVDs in New Mexico is

$$C = 80{,}000 \text{ dollars} + \frac{3 \text{ dollars}}{20 \text{ DVDs}} \bullet \times \text{ DVDs}$$

$$C = \left( 80{,}000 + \frac{3}{20} \times \right) \text{dollars}$$

$$C = (80{,}000 + .15\times) \text{ dollars}$$

---

e. Find an equation for the cost of making *x* number of DVDs in the factory in Colorado.

**Solution:**

e. The cost *C* for making $x$ DVDs in Colorado is

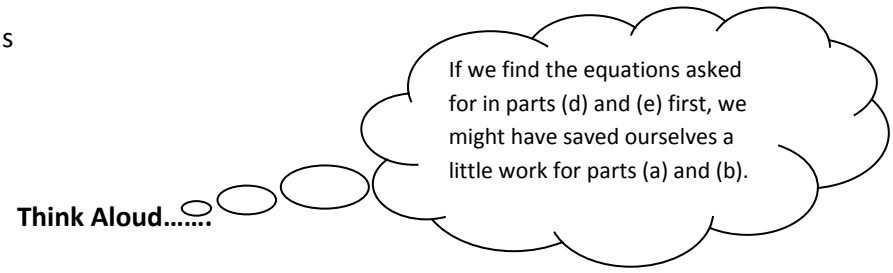
$$C = 100{,}000 \text{ dollars} + \frac{1.5 \text{ dollars}}{10 \text{ DVDs}} \bullet x \text{ DVDs}$$

$$C = \left(100{,}000 + \frac{1.5}{10}x\right) \text{ dollars}$$

$$C = (100{,}000 + .15x) \text{ dollars}$$

---

**Solution II:** Working Backwards




If we make $x$ DVDs and we want the unit cost per DVD, we could just use the equation for the cost for $x$ DVDs in New Mexico:

$$\frac{(80{,}000 + .15x) \text{ dollars}}{x \text{ DVDs}} = \left(\frac{80{,}000}{x} + .15\right) \frac{\text{dollars}}{\text{DVD}}$$

Or, in terms of cents:

$$\left(\frac{80{,}000}{x} + 1.5\right) \frac{\text{dollars}}{\text{DVD}} \times \frac{100 \text{ cents}}{\text{dollar}} = \left(\frac{8{,}000{,}000}{x} + 15\right) \frac{\text{cents}}{\text{DVD}}$$

Now, to do part (a), we just plug in 1,000,000 and we get that one DVD costs (8+15) cents = 23 cents.

If we make $x$ DVDs and we want the unit cost per DVD in Colorado, we could use the equation for the cost for $x$ DVDs in Colorado:

$$\frac{(100{,}000+.15x) \text{ dollars}}{x \text{ DVDs}} = \left(\frac{100{,}000}{x} + 1.5\right) \frac{\text{dollars}}{\text{DVD}}$$

Or, in terms of cents:

$$\left(\frac{100{,}000}{x} + 1.5\right) \frac{\text{dollars}}{\text{DVD}} \times \frac{100 \text{ cents}}{\text{dollar}} = \left(\frac{10{,}000{,}000}{x} + 15\right) \frac{\text{cents}}{\text{DVD}}$$

Now, to do part (b), we just plug in 1,000,000 and we get that one DVD costs (10+15) cents = 25 cents.

# Project #16

**Domain:** Statistics and Probability
**Standard SP8**: Investigate patterns of association in bivariate data.

**Directions:** The table below shows test scores for a class.
A stem and leaf plot is a method of graphing a collection of numbers by placing the "stem" digits (or initial digits) in one column and the "leaf" digits (or remaining digits) out to the right.

| Stem | Leaf |
|---|---|
| 9 | 0 1 1 5 7 |
| 8 | 0 0 2 4 6 7 9 |
| 7 | 7 7 8 9 |
| 6 | 9 |
| 5 | 2 3 |
| 4 | 4 |

**Solution:**
**Review Stem and Leaf Graphs:**
The "stem" is the left-hand column which contains the tens digits. The "leaves" are the lists in the right-hand column, showing all the ones digits for each of the tens, twenties, thirties, and forties. That's pretty much all there is to a stem-and-leaf plot. You're just listing out how many entries you have in certain classes of numbers, and what those entries are.

---

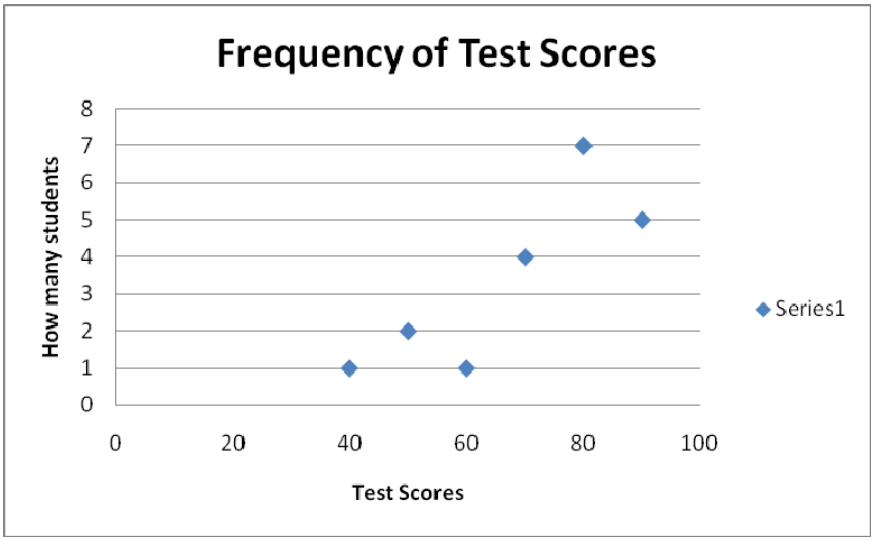
1. How many students scored in the 80's?

**Solution:**
There are 7 students who scored in the 80's.

Now, show the same data represented in another graphic form (i.e., bar, line graph, scatter plot or pie graph).

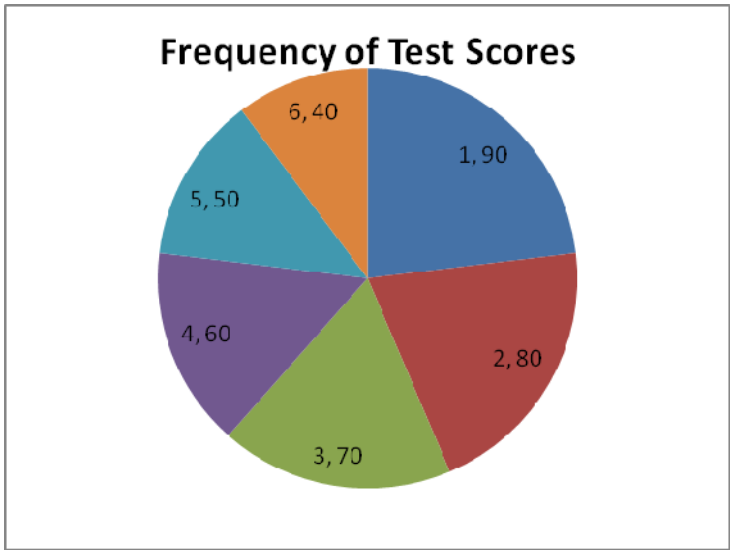
**Solution:**

**Scatter Plot**




**Pie Graph**

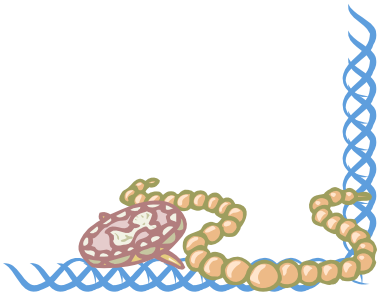
| Score | # of students |
|---|---|
| 90 | 5 |
| 80 | 7 |
| 70 | 4 |
| 60 | 1 |
| 50 | 2 |
| 40 | 1 |



# Project #17

**Domain:** Statistics and Probability

**Standard SP8:** Investigate patterns of association in bivariate data.



**Directions:** Drawing from a set of **six** blue beads and **three** gold beads, use ratios to state the likelihood of each color being drawn; conducts experiments to test predictions. Show your work and explain your answer.

---

**Solution:** Use the information given to solve the problem

Step 1: Add 6 + 3 to get the total number of beads

Step 2: Figure out each color beads ratios
Blue = 6:9 or 2:3
Gold = 3:9 or 1:3

# Project #18

**Domain:** Statistics and Probability
**Standard SP8:** Investigate patterns of association in bivariate data.

**Directions:** Design experiments to answer class or personal questions, collect information, and interpret the results.

1) Create a question that you want to survey people to find the answer. (Example: What is your favorite football team?)
2) Ask at least 10 people to answer the question.
3) Chart the answers to your question in a graph form that you think would be the best way to display the information: pictograph, bar graph, circle graph, or line graph.

---

**Solution:**

Answers will vary

**Rubric:**

+ Must have a survey question relevant to something you have studied this year.
+ Must have at least 10 subjects (people that you surveyed)
+ Must have selected the best way (graph or chart) to represent your data. Remember, graphs have certain characteristics that lend their use to certain types of data.

**Line graph:** Line graphs are used to track changes over short and long periods of time. When smaller changes exist, line graphs are better to use than bar graphs. Line graphs can also be used to compare changes over the same period of time for more than one group.
**Pie Chart:** Pie charts are best to use when you are trying to compare parts of a whole. They do not show changes over time.
**Bar Graph:** Bar graphs are used to compare things between different groups or to track changes over time. However, when trying to measure change over time, bar graphs are best when the changes are larger.
**Area Graph:** Area graphs are very similar to line graphs. They can be used to track changes over time for one or more groups. Area graphs are good to use when you are tracking the changes in two or more related groups that make up one whole category (for example public and private groups).
**X-Y Plot:** X-Y plots are used to determine relationships between the two different things. The x-axis is used to measure one event (or variable) and the y-axis is used to measure the other. If both variables increase at the same time, they have a positive relationship. If one variable decreases while the other increases, they have a negative relationship. Sometimes the variables don't follow any pattern and have no relationship.

# Project #19

**Domain:** Statistics and Probability Number System
**Standard EE8:** Understand the connections between proportional relationships, lines, and linear equations
**Imagine Subject Area:** Financial Literacy

**Directions:** College costs increase at about twice the inflation rate. Current increases have averaged 5% to 8% annually.

According to the College Board's and Trends in College Pricing, the 2011-2012 average total costs (including tuition, fees, room and board) were **$16,140** for students attending four-year public colleges and universities **in-state** and **$28,130** **out-of-state**, and **$36,993** for students at four-year **private** colleges and universities. You can assume an additional **$4,000** for textbooks, supplies, transportation and other expenses.

Use the information above to **estimate** the cost of your freshman year of college based on a yearly increase in costs of **5%** for the **next five years**. Record your work below.

---

**Solution: Multi-step Problem – make a chart**

Step 1: Add your textbooks, supplies, transportation and other to the tuition
Step 2: Convert 5% into a decimal .05 for computations
Step 3: Multiply .05 to your total (round this off to the nearest dollar.
Step 4: Add this increase to the total
Step 4: Now multiply the new total by .05
Step 5: Continue this process five times.
Step 6: Write the answers in the chart and then go back and answer the question directly.

**Think Aloud**

This is a real life situation, granted we are assuming you will be going to college in five years, so we will be multiplying the total by .05 five times.

| University | 2012-2013 | | 2013-2014 | | 2014-2015 | | 2015-2016 | | College |
|---|---|---|---|---|---|---|---|---|---|
| | Tuition | + .05 | Tuition | + .05 | Tuition | + .05 | Tuition | +.05 | Your Cost in 2016 |
| **In-state** | $20,140 | $1,007 | $21,147 | $1,058 | $22,205 | $1110 | $23,315 | $1166 | $24,481 |
| **Out-Of-State** | $32,130 | $1,6007 | $33,737 | $1,687 | $35,424 | $1,771 | $37,195 | $1,898 | $39,093 |
| **Private** | $40,993 | $2,049 | $43,042 | $2,152 | $47,194 | $2,360 | $49,554 | $2,478 | $52,032 |

**Answer:**

In-state = $24,481.00
Out-of-state = $39,093.00
Private = $52,032.00

# Project # 20

**Domain:** Number System
**Imagine Subject Area**: Financial Literacy
**Standards:** Various

**Directions:** Create a personal monthly budget for yourself. Assume you are 10 or 15 years older than you are now and are employed full time. Create the budget on paper or using a computer program like Excel. Create a list of expenses and how much you think each would cost monthly. Consider expenses such as college loans, car payments, insurance, rent, phone, water, cable, electricity, etc. Feel free to include additional expenses such as gifts, professional costs (such as certification or graduate classes), or cost of a wedding or children. Assume your personal income is $2,000 a month. Then, after you create your budget, discuss the budget with an adult to determine whether or not your projections are accurate.

---

**Solution:**

Answers may vary

**Rubric:**

+ Must be in an excel file
+ Must have at least 8 expenses
+ Must have an adult sign off on it.

| EXPENSES | AMOUNT | REVENUE |
|---|---|---|
| | | **$2,000.00** |
| Rent/Mortgage | $700.00 | |
| Food | $100.00 | |
| Electricity | $50.00 | |
| Gas/Transportation | $50.00 | |
| Phone | $20.00 | |
| Car payment | $100.00 | |
| Cable | $20.00 | |
| Car Insurance | $40.00 | |
| Health Insurance | $30.00 | |
| Water | $10.00 | |
| College Loan | $75.00 | |
| **TOTAL EXPENSES** | $1,195.00 | **$1,195.00** |
| | | **$805.00** |