Imagine Schools
Summer Math Challenge

Grade Eight

Dear Imagine Student,

We hope you will enjoy this Math Challenge Packet and work hard to complete all problems on your own or with help from a parent or guardian. All projects in the Challenge Packet are based on the Common Core State Standards. Therefore, this should be a review for you in some ways, but should stretch you as you apply your understanding of concepts you learned throughout this past year. We suggest doing one project each day. Once you have finished with the project you select for the day, try to find a way to discuss it with a friend, parent, or relative. Think about how the skills and concepts in the problem you completed are connected to other things in your home, environment, or daily routine. Find ways to apply your new understanding to real world situations.

Math is all about problem solving. One of the best ways to learn math is to try out problems in which you have to devise your own strategy to find the solution. There is usually more than one way to solve math problems. While working on the problems in this packet, you may discover shortcuts and use your own process or set of rules to calculate or determine the appropriate solution. Make sure to keep notes, include your work so you can justify your solutions. In other words, be sure you can answer the question, "How do you know?" Explaining how you arrived at an answer immediately tells others what you're learning along the way.

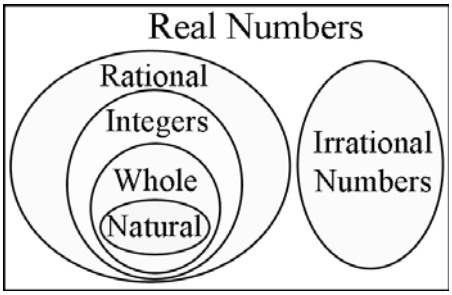
Sincerely,
Imagine Schools National Academic & Character Team

Project #1

Domain: Number System

Standard: NS8: Know that there are numbers that are not rational, and approximate them by rational numbers.

Directions: Pretend your 10-year-old younger brother or sister knows nothing about integers, rational, whole, natural, and irrational numbers. Write two to three paragraphs explaining the graph below. Be sure to use language that a fourth or fifth grader can understand.



Vocabulary	Definitions
Real Numbers	All the numbers that can be represented by points on a number line.
Rational	All numbers that can be written as a ratio of two integers. (15/16 and 37)
Integers	The set of counting numbers, their opposites, and zero; (-2, -1, 0, 1, 2….) NOT 15/8, -0.98
Whole	The members of a set (1,2,3,4) NOT -3, 0.56, 100 ¾
Natural	Or counting numbers, numbers that are used to count (1,2,3,4…)
Irrational	Numbers that cannot be expressed as a ratio of two integers. (π)

Project # 2

Domain: Number System

Standard NS8: Know that there are numbers that are not rational, and approximate them by rational numbers.

Directions: Compare each pair of numbers using <, >, or =. Write one to two sentences after each problem explaining your answer.

- $-\frac{11}{21} \square -\frac{13}{21}$

- $-\frac{7}{5} \square$ -1.35

- $2\frac{3}{4} \square$ 2.75

Now, create two to three similar problems and challenge an adult or friend to solve them.

Project #3

Domain: Number System
Standard NS8: Know that there are numbers that are not rational, and approximate them by rational numbers.

Directions: List the numbers $\frac{2}{3}$, $-\frac{2}{3}$, 1.2, $\frac{4}{3}$, $-\frac{4}{3}$, -1.2, and $-\frac{7}{4}$ from least to greatest and then locate the numbers on the number line. You will label the number line below to fit your purposes.

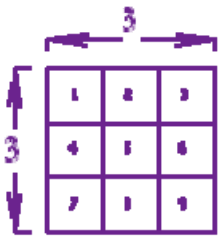
Project #4

Domain: Number System
Standard NS8: Know that there are numbers that are not rational, and approximate them by rational numbers.

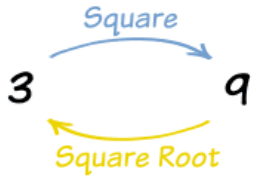
Directions:
Without using the square root button on your calculator, estimate $\sqrt{800}$ as accurately as possible to 2 decimal places. (It is worth noting that $20^2=400$ and $30^2=900$.)

Review:
(Hint: Review the meaning of Square and Square Root.)

How to Square A Number
To square a number, just multiply it by itself ...
Example: What is 3 squared? 3 x 3 = 9 or 3^2



Square Roots
A **square root** goes the other way:



3 squared is 9, so a **square root of 9 is 3**

A square root of a number is ... a value that can be **multiplied by itself** to give the original number.
A square root of **9** is ... **3**, because **when 3 is multiplied by itself** you get **9**.
It is like asking: What can I multiply by itself to get this?

To help you remember think of the root of a tree:
"I know the tree, but what is the root that produced it?"
In this case the tree is "9", and the root is "3".

Project #5

Domain: Functions
Standard F8: Use functions to model relationships between quantities.

Directions:
Solve the problem below.

Linda traveled 110 miles in 2 hours. If her speed remains constant, how many miles can she expect to travel in 4.5 hours? Answer the question in complete sentences and show your work.

Then, create a problem of your own involving linear functions and constant rates of speed over a specified distance.

Project # 6

Domain: Functions
Standard F8: Define, evaluate, and compare functions.

The table shows a relation between x and y.

x	2	3	4	5
y	7	10	13	16

Which of these equations expresses this relation? Explain your answer in complete sentences.

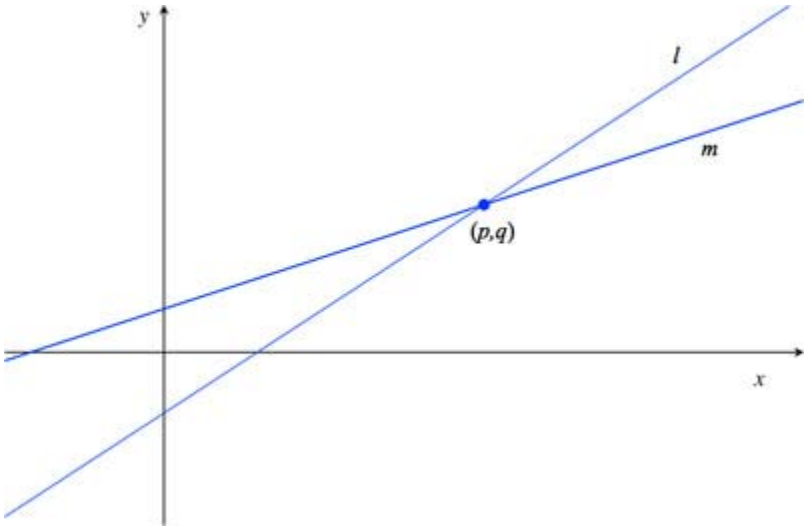
A. $y = x + 5$
B. $y = x \pm 5$
C. $y = \frac{1}{3}(x \pm 1)$
D. $y = 3x + 1$

How might you use information like the above in the real world? Describe a situation in which you would need to create a table and apply your understanding of a function.

Project #7

Domain: Expressions and Equations
Standard EE8: Understand the connections between proportional relationships, lines, and linear equations.

Directions:
The figure below shows the lines l and m described by the equations $4x-y=a$ and $y=2x+b$, for some constants a and b. They intersect at the point (p, q).



1. How can you interpret a and b in terms of the graphs of the equations above?"

2. Imagine you place the tip of your pencil at point (p, q) and trace line l out to the point with x -coordinate p + 2. Imagine I do the same on line m. How much greater would the y -coordinate of your ending point be than mine?

Now, create a function of your own. State the coordinates of two points along the line. Challenge a friend or a parent to find the slope. *Keep in mind that you may need to teach them how to find the slope before you ask them to solve your problem. You are the expert now!

Project #8

Domain: Functions
Standard F8: Use functions to model relationships between quantities.

Which of the following could be modeled by $y=2x+5$? Answer YES or NO for each one.

a. There are initially 5 rabbits on the farm. Each month thereafter the number of rabbits is 2 times the number in the month before. How many rabbits are there after *x* months?
   YES NO

b. Joaquin earns $2.00 for each magazine sale. Each time he sells a magazine he also gets a five-dollar tip. How much money will he earn after selling *x* magazines?
   YES NO

c. Sandy charges $2.00 an hour for babysitting. Parents are charged $5.00 if they arrive home later than scheduled. Assuming the parents arrived late, how much money does she earn for *x* hours?
   YES NO

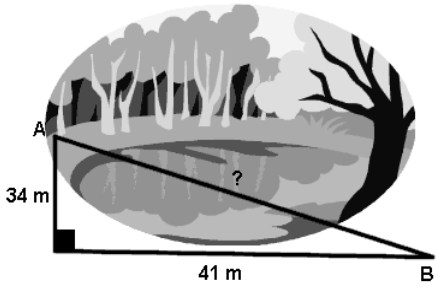
d. I have a sequence of integers. The first term of the sequence is 7 and the difference between any consecutive terms is always equal to 2.
   YES NO

e. Sneak Preview is a members-only video rental store. There is a $2.00 initiation fee and a $5.00 per video rental fee. How much would John owe on his first visit if he becomes a member and rents *x* videos?
   YES NO

f. Andy is saving money for a new CD player. He began saving with a $5.00 gift and will continue to save $2.00 each week. How much money will he have saved at the end of *x* weeks?
   YES NO

Project # 9

Strand: Geometry
Standard G8: Understand and apply the Pythagorean Theorem.



The Pythagorean Theorem is $a^2 + b^2 = c^2$. C is the side opposite the right angle. In this particular problem "C" is the unknown side.

Pythagorean Theorem in the Real-World

To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond? Choose: 22, 34, 53, or 75. Show all of your work and explain how you came up with your solution.

Project #10

Domain: Number System
Standard NS8: Know that there are numbers that are not rational, and approximate them by rational numbers.
Imagine Subject Area: Financial Literacy

Directions: The United States owes approximately $14,300,000,000,000.00 in National debt. There are approximately 300 million people in the United States. Assume the United States would like to pay off the debt in one lump sum. How much would each American have to pay to pay off the National debt?

If each American paid $2,000 a year towards the National debt how many years would it take for the country to pay off what it owes? (Assume that there is no interest on the debt.)

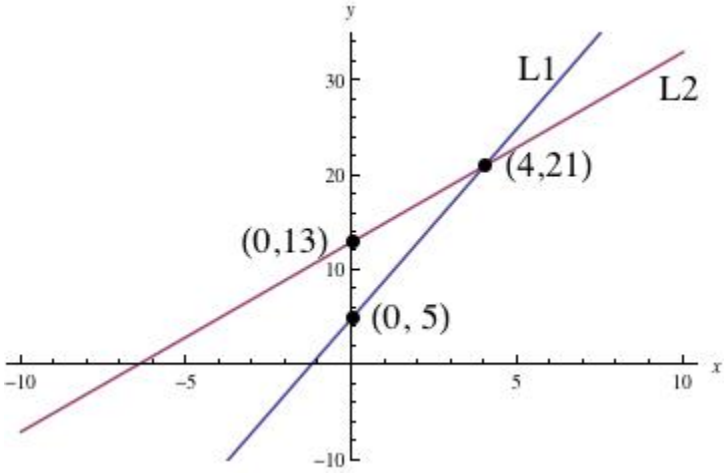
Project # 11

Domain: Equations and Expressions

Standard EE8: Analyze and solve linear equations and pairs of simultaneous linear equations

Directions:

Consider the graph below showing two lines, L1 and L2



1. Find the two corresponding linear equations.
2. Find points other than the ones given in the graph; one that lies on L1 but not on L2 and one that lies on L2 but not on L1.
3. Name three examples of congruent angles in nature, at school or in the gymnasium.

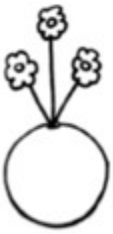
Project #12

Domain: Geometry
Standard G8: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

My sister's birthday is in a few weeks and I would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and I want to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.

Cylinder Vase
Show off your flowers in this beautiful vase.
10cm X 40 cm
$9.95
4KE09

Cone Vase
This vase holds your flowers in place!
16cm X 45cm
$9.95
4KE08



Sphere Vase
Doesn't get any more symmetric than this!
18cm X 18cm
$9.95
4KE07

Pencil Vase
This perfect gift for your math teacher!
12cm X 42cm
$9.95
4KE06

1. Which vase should I purchase?
2. How much more water does the largest vase hold than the smallest vase?
3. Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?
4. The vase company designs a new vase that is shaped like a cylinder on bottom and a cone on top. The catalog states that the width is 12 cm and the total height is 42 cm. What would the height of the cylinder part have to be in order for the total volume to be 1224π cm^3?
5. Design your own vase with composite shapes, determine the volume, and write an ad for the catalog.

Project #13

Domain: Geometry
Standard G8: Understand congruence and similarity using physical models, transparencies, or geometry software.

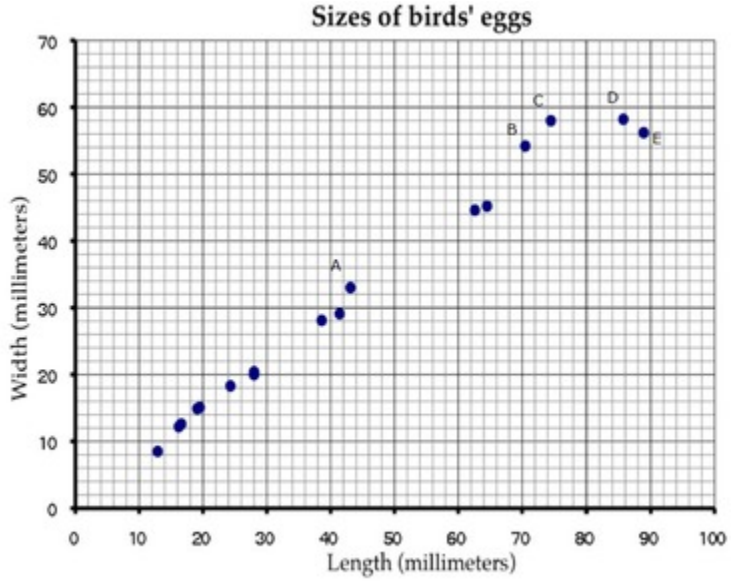
Research and compare the speed, height, track length, and duration of ride for two roller coasters. Choose two pictures of roller coasters from the web site www.joyrides.com (Vortex and King Cobra, for example). Then do research online to find the answers to the questions below. Be sure to include the proper units after each measurement.

Coasters		
	Name of Coaster:	**Name of Coaster:**
Speed		
Height		
Track Length		
Duration		

Project #14

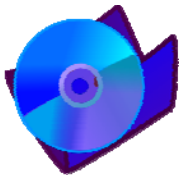
Domain: Statistics and Probability
Standard: Investigate patterns of association in bivariate data.

Directions: Read the information below and then solve the problem.
This scatter diagram shows the lengths and widths of the eggs of some American birds.



a. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.
b. What does the graph show about the relationship between the lengths of birds' eggs and their widths?
c. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?
d. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.
e. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

Project #15



Domain: Expressions and Equations

Standard EE8: Understand the connections between proportional relationships, lines, and linear equations.

Directions:

Read the problems carefully and solve each problem a-e

a. DVDs can be made in a factory in New Mexico at the rate of 20 DVDs per $3, but the factory costs $80,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?

b. DVDs can be made in a factory in Colorado at the rate of 10 DVDs per $1.50, but the factory costs $100,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?

c. How much can a buyer save on a million DVDs by buying DVDs from New Mexico instead of DVDs from Colorado?

d. Find an equation for the cost of making *x* number of DVDs in the factory in New Mexico.

e. Find an equation for the cost of making *x* number of DVDs in the factory in Colorado

Project #16

Domain: Statistics and Probability
Standard SP8: Investigate patterns of association in bivariate data.

Directions: The table below shows test scores for a class.
A stem and leaf plot is a method of graphing a collection of numbers by placing the "stem" digits (or initial digits) in one column and the "leaf" digits (or remaining digits) out to the right.

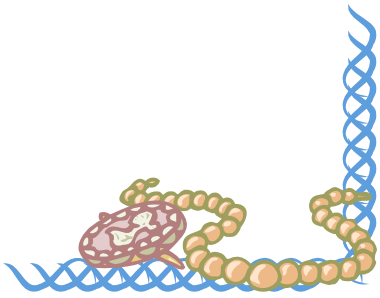
Stem	Leaf
9	0 1 1 5 7
8	0 0 2 4 6 7 9
7	7 7 8 9
6	9
5	2 3
4	4

1. How many students scored in the 80's?
2. Now, show the same data represented in another graphic form (i.e., bar, line graph, scatter plot or pie graph).

Project #17

Domain: Statistics and Probability

Standard SP8: Investigate patterns of association in bivariate data.



Directions: Drawing from a set of six blue beads and three gold beads, use ratios to state the likelihood of each color being drawn; conducts experiments to test predictions. Show your work and explain your answer.

Project #18

Domain: Statistics and Probability
Standard SP8: Investigate patterns of association in bivariate data.

Directions: Design experiments to answer class or personal questions, collect information, and interpret the results.

1) Create a question that you want to survey people to find the answer. (Example: What is your favorite football team?)
2) Ask at least 10 people to answer the question.
3) Chart the answers to your question in a graph form that you think would be the best way to display the information: pictograph, bar graph, circle graph, or line graph.

Project #19

Domain: Statistics and Probability, Number System
Standard EE8: Understand the connections between proportional relationships, lines, and linear equations.
Imagine Subject Area: Financial Literacy

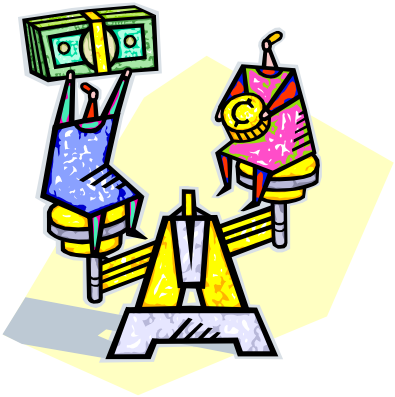
Directions: College costs increase at about twice the inflation rate. Current increases have averaged 5% to 8% annually.

According to the College Board's and Trends in College Pricing, the 2010-2011 average total costs (including tuition, fees, room and board) were $16,140 for students attending four-year public colleges and universities in-state and $28,130 out-of-state, and $36,993 for students at four-year private colleges and universities. You can assume an additional $4,000 for textbooks, supplies, transportation and other expenses.

Use the information above to calculate the cost of your freshman year of college based on a yearly increase in costs of 5% for the next four years. Record your work below.

Project # 20

Domain: Number System
Imagine Subject Area: Financial Literacy
Standards: Various



Directions: Create a personal monthly budget for yourself. Assume you are 10 or 15 years older than you are now and are employed full time. Create the budget on paper or using a computer program like Excel. Create a list of expenses and how much you think each would cost monthly. Consider expenses such as college loans, car payments, insurance, rent, phone, water, cable, electricity, etc. Feel free to include additional expenses such as gifts, professional costs (such as certification or graduate classes), or cost of a wedding or children. Assume your personal income is $2,000 a month. Then, after you create your budget, discuss the budget with an adult to determine whether or not your projections are accurate.